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Original paper

A few considerations of climbing-snail problem: Fibonacci's error, problem's popularity and Mexican students' performances

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Extended summary

Among the "puzzle-like" problems of "recreational mathematics" is very popular one related to the situation in which a snail moving up and down on a pole of certain height. The question usually read: How many days are needed that the snail reaches the top of the pole? People often give the wrong answer to this question because they use "fast thinking" that does not take into account the important fact that the snail should get to the top during a daily climbing and not during a night descending (explicit or implicit conceptual error that can be called "flying snail "). At the beginning, we briefly present (mostly unknown) history of this problem, which shows that the wrong solutions to the problem were given some famous medieval mathematicians such as Fibonacci, Calendri and Ries.

After that, we give some concrete examples that show a great popularity of the problem on web pages with puzzles, in books with mathematical riddles, in mathematics textbooks and manuals for preparing entrance exams at universities. Despite of such big popularity, concrete pupils' and students solutions of the problem of snail that climbs and goes down are insufficiently present in the research literature. In this way, phenomenological categorization of these solutions and theoretical considerations of their possible causes are missing.

In the main part of the work, we describe an initial study conducted in Mexico, which his the first contribution to a phenomenological categorization of different (correct and incorrect) students' solutions and to a consideration of possible causal factors of their diversity. The aim of the study was to qualitatively investigate how students' age and position the correct answer in a multiple choice for math influence students' solutions. The pupils of different ages

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(12 and 15) were given two versions of the problem. In one group, the correct answer was the first offered choice (N1 = 68), while in the second group the correct answer was the last offered choice (N2 = 81).

Obtained results show that the students' solutions were affected both by their age and by position the correct answer among the offered answers. When the correct answer was in the first place, older students (15 years) have significantly out performed the younger (12 years). However, when the correct answer was in the last place, the older students were only slightly better.

Various procedures and representations used by students to come to a right or wrong answer are presented with details. Finally, we mention several implications of the obtained results that are important for teaching of mathematics: (1) Teachers should avoid, when ever possible, to give learners problems in multiple-choice format. (2) Teachers should constantly ask students to describe verbally what they do in solving problems and why they do it. (3) Teachers should constantly encourage students to find many different representations and procedures for obtaining a solution to all mathematical problems they face (for example, verbal, tabular or schematic approach). (4) Teachers should explicitly show to students the essential difference between the pictorial and schematic representation of the problem situation and give students multiple opportunities to move from the pictoric to the schematic representations, first with teacher's help and later without this help. (5) Teachers should show students the importance of taking into account units of physical quantities in solving problems related to physics.

Key words: Climbing-snail problem, Fibonacci's error, mathematical problem solving, multi¬ple-choice format, students' drawings, students' mathematical reasoning

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