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Original paper

Sum and Difference Constancy Rules as a Calculus Strategy Base[2]

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Extended summary

Developing skills of calculating is one of the primary tasks of initial Arithmetic teaching. Fulfilling calculating skills is one of the basic aims of initial Arithmetic teaching. Arithmetical operations and calculating algorithm present a greatest part of the primary school Mathematics curriculum throughout the world. Standard algorithms and alternative strategies divide a complex task of calculating into the steps, in the forms of simpler tasks upon which operations are done with available knowledge of procedures. Using only one procedure in calculating prevents development of mathematics literacy in children. The most convenient strategy of calculating depends on the characteristics of numbers in an expression, students' age and situation in which it is being calculating. Students who have a high level of comprehension of the relation between addition and subtraction, characteristic of the decade number system and rules which describe them (arithmetic rules) show greater flexibility in choosing adequate strategies of calculating. The basis of different strategies and procedures of calculating are the mathematics rules. Fuson and his associates characterized using arithmetical rules for making the strategies of calculating as "theorems in action" (Fusion at al., 1997: 150). Although students calculate successfully, and transform expressions and complex procedures of calculation, they know isolated rules of arithmetic and this kind of knowledge often does not represent a coherent system, which is the feature of fundamental comprehension. One of the ways to overcome the stated problem is that students should be more involved into the process of generalization of knowledge, and that their attention should be focused on the terms and structures, which represent significant rules of arithmetic, and the calculating procedures based upon them. These

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2 This paper is, in a certain extent, an extended part of the Master thesis Comprehension and application of Sum and Difference Constancy Rules as a Calculus Strategy Base, defended at the Teacher Education Faculty, University of Belgrade (mentor Marijana Zeljić, PhD).

kinds of activities must be led with definite aim of explicit abstraction of arithmetic structures. In our educational praxis, sum and difference constancy rules rules, are being systematically studied in the fourth grade and this is seen through the three types of expression (procedural, rhetorical and symbolic), whereas in foreign reference (Carpenter at al., 1998; Fuson at al., 1997; Torbeyns at al., 2009) the stated rules are treated as strategies in calculating, without explicit exception and presenting of this rule. Strategies of calculating based on the sum and difference constancy rules are called the compensation strategy.

The aim of the completed research is studying the degree of comprehension of the sum and difference constancy rule by the students, and comprehension is seen as and operative application of the stated rules in the procedures of calculating. Based on this aim, the following tasks of the Reserch follow:

- Studying whether all knowledge components (rhetorical and symbolic expression of rules as well as application of rules as strategies in calculating in different situations) are equally developed.

- Studying whether students use rules as a strategy for calculating in different examples (in the test paper/pen).

- Studying whether students use rules as mental strategy of calculating (in mind).

The sample of the research consists of 39 fourth grade students of a primary school. Descriptive method was used as well as techniques of testing and interviewing (structured conversation with students). The knowledge test and the flow of conversation with students, i.e. connected interview were composed for the purposes of this research. The test had the aim of checking whether students could express the rules rhetorically and symbolically and whether they use the strategy of compensation (procedural expressing) when they have pen and paper and with an interview when they do not have them.

Results of the research show that students show statistically best achievements in rhetorical expression of rules, and those they achieve worse results in generalisation of rules, and the worst results are shown in operational application of rules. Strategies based on the stated rules are more often applied as mental strategies than in the situation paper-pen, but the analysis of the results show that students are not flexible in choosing the appropriate strategy. In the paper in which paper and pen were allowed, the students used standard strategies and they were successful but we can also ask a question of adequacy of the applied strategies. Success of using the standard procedures significantly falls when they must be calculated without a paper and pen and third proves that students have a suitable degree of procedural fluency. Examples used in interviewing students are typical examples in which compensation strategy (sum and difference constancy rule) is the most suitable and the examples are: 348 + 199, 292 + 548, 823 – 298, 676 – 305. Slow, inefficient calculating becomes an obstacle for understanding different topics, because the focus is changed from tasks to calculating. Recognising and using different strategies presents aid in calculating, and its significance is in better understanding of the expression structure, arithmetical rules and their application, and this contributes to better understanding of the characteristics of the decade system,opertions and arithmetic itself.

Implications of this research are seen in concretisation of methodological instructions in the curriculum and this would represent clearer directions for both teachers and course book authors. Requirement for developing different strategies of calculating should be clearly implemented in the curriculum. Changes in the curricula and course books are the bases for developing methodological approaches of teachers, which stimulate understanding and adequate application of rules as easier points in calculating by the students, and this leads to developing diffeenrent strategies and development of mathematical literacy.

Key words: Sum and difference compensation strategy, calculating strategies.

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