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Original paper

Prospective Primary School Teachers'and Pre-school Teachers' Beliefs about the Nature of Mathematics and Mathematics Learning[2]

Summary: University studies, among other things, are aimed at enabling primary school teachers and pre-school teachers to support the development of mathematical competences of young learners. Although mathematics content knowledge is a major component of the professional body of knowledge required for teaching mathematics, teachers' professional beliefs on what mathematics is and how mathematics is learned have a significant mediating effect on teachers' success in providing genuine opportunities for learning meaningful mathematics.

The research goal of the study conducted at the beginning of the second semester, when students encounter their first mathematics course for teachers, was to analyze the prospective teachers' beliefs on the nature of mathematics and on mathematics learning. The student questionnaire consisted of parts of the questionnaire used in the international study TEDS-M and of a small number of mathematics items designed to verify the answers given by the questionnaire respondents. The results revealed a difference between the self-professed beliefs of the students and the approaches they used to respond to the mathematics items. These findings point to the need for providing specific learning opportunities within initial teacher education to help future teachers in developing coherent mathematical knowledge for teaching and consistent professional beliefs.

Keywords: mathematics education, primary school teachers, pre-school teachers, professional knowledge, beliefs.

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Introduction

Learning, in general, and learning mathematics, in particular, is a complex process. Consequently, teaching, in general, and teaching mathematics, in particular, is a complex endeavor. Research literature on teacher education is abundant with attempts to define, analyze and characterize the most important competencies required for teaching mathematics. However, it "lacks a common theoretical basis, which prevents a convincing development of instruments and makes it difficult to connect studies to each other" (Blömeke, Felbrich, Müller, Kaiser &Lehmann, 2008, as cited in Blömeke & Delaney, 2012, p. 223). Based on a considerable body of knowledge published by various authors like Blömeke& Paine (2008), Ferrini-Mundy, Floden, McCrory, Burrill, & Sandow (2005), Lester (Ed.) (2007), Schoenfeld (2011), Schoenfeld,& Kilpatrick (2008), Shulman (1985, 1987), Richardson (1996), Rowland, & Ruthven (Eds.), (2010), Thompson (1992) and others in the last several decades, as well as on the knowledge accumulated within the work of several projects:

– "Mathematics Teaching in the 21st Century (MT21)" (Schmidt, Blömeke,& Tatto, 2011; Schmidt,Houang, Cogan, Blömeke, Tatto, Hsieh, et al. 2008; Schmidt, Tatto, Bankov, Blömeke, Cedillo, Cogan, et al. 2007);
– "Teacher Education and Development Study in Mathematics (TEDS-M)" (Tatto, Peck, Schville, Bankov, Senk, Rodriguez, Ingvarson, Reckase, &Rowley, 2012; Tatto, Senk, Rowley, & Peck, 2011; Tatto, Schville, Senk, Ingvarson, Peck, & Rowley, 2008); and
– "Learning mathematics for teaching (LMT)"(Hill, Ball, & Schilling, 2008; Hill, Schilling, & Ball, 2004; Hill, Sleep, Lewis, & Ball, 2007);

Blömeke & Delaney (2012) have presented a conceptual framework of teacher competencies (Figure 1) widely accepted in the mathematics education research community.

As elaborated by Schoenfeld (2010) in his book "*How We Think*", teaching is a "well-practiced, knowledge-intensive domain" in which teachers' decision making is "a function of their orientations, resources, and goals" (p. 187), with mathematics knowledge being the most important resource for teachers of mathematics (which generalist primary teachers are). Teachers' professional beliefs, motivation and self-regulation have a direct influence on how teachers access this knowledge base in their instructional practices.

A comprehensive review of the research on mathematics teachers' beliefs and affects, presenting the commonly accepted definitions of affect, beliefs, attitudes, and emotions, is given in Chapter 7 by Phillip (2007) in the *Second Handbook of Research in Mathematics Teaching and Learning* (Lester (Ed.),
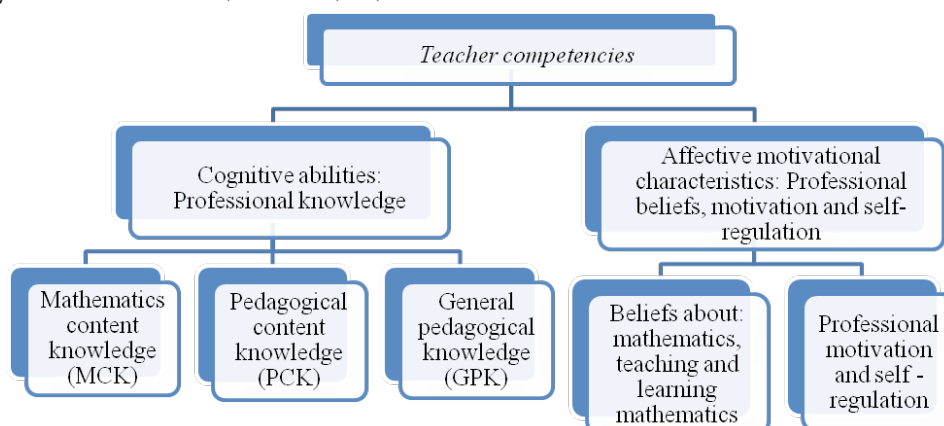

Figure 1 – Conceptual framework of teacher competencies

2007). *Affect* is"a disposition or tendency or an emotion or feeling attached to an idea or object" (Phillip, 2007, p.259). It is comprised of: *beliefs* ("psychologically held understandings, premises, or propositions about the world that are thought to be true"), a*ttitudes* ("manners of acting, feeling or thinking that show one's disposition or opinion"), and *emotions* ("feelings or states of consciousness, distinguished from cognition") (Phillip, 2007, p.259). Beliefs are considered as lenses which "filter some complexity of a situation to make it comprehensible, shaping individuals' interpretations of events" (Grant, Hiebert, & Wearne, 1998, as cited by Phillip, 2007, p.270). Beliefs influence perception and predispose toward action; they develop gradually, and cultural factors play a key role in their development; some beliefs (*primary*) serve as the foundation of other beliefs (*derivative*) in a quasilogical structure; *central* beliefs are held strongly, *peripheral* beliefs are more susceptible to change; beliefs are held in clusters relatively isolated from other clusters; belief systems may appear contradictory or inconsistent to an observer, as they are context specific and situated (Phillip, 2007).

Studying teachers' beliefs and knowledge is motivated by the notion that teachers' beliefs and knowledge shape teachers actions (Lester (Ed.), 2007; Rowland, & Ruthven (Eds.), 2010; Stipek, Givvin, Salmon, &MacGyvers, 2001). Research studies demonstrate that teachers' practices are consistent with teachers' beliefs about mathematics to a higher degree than with teachers' beliefs about teaching and learning (Phillip, 2007), and that teachers' practices impact students' development of mathematics proficiency (Darling-Hammond, 2000; Hill, Ball, Blunk, Goffney, & Rowan, 2007, Hill, Rowan, & Ball, 2005; Tatto, et al., 2012).

Method

The research reported in this paper is a preliminary study conducted before the start of a larger national project "*The Beliefs of Prospective Primary School Teachers and Pre-School Teachers about the Nature of Language and of Mathematics and about Teaching and Learning Language and Mathematics*", carried out by a multidisciplinary research team from the Faculty of Pedagogy "St. Kliment Ohridski" in Skopje and funded by the Cyril & Methodius University – Skopje. The goal of the national project was to gather empirical data regarding the broader belief systems of the future primary school and pre-school teachers in the final (8th) semester of their university studies (after the completion of all the sequences of courses and practical training in schools and pre-school institutions) about the (native) language and literature and about mathematics, as well as about teaching and learning language, literature, and mathematics. The sample of respondents in the national project consisted of students from each of the four higher education institutions in the country offering university programs in the primary and pre-school teacher education.

The sample of the respondents for the preparatory study consisted of 102 first-year students (89 female, 87 % of the sample) enrolled in the first of a sequence of compulsory mathematics and mathematics methodology courses for primary school teachers (71 respondents) and preschool teachers (31 respondent); with 87 students following instruction in Macedonian language, and 15 students in Turkish language. The survey consisted of the beliefs scales related to mathematics, mathematics learning and mathematics achievement developed in TEDS-M (Tatto, et al., 2012), and incorporated in the survey questionnaire which was later used in the national project. The survey was administered during the first week of the second semester of university studies (at the beginning of the semester when the students encounter university mathematics instruction for the first time). Additionally, a sub-sample of 71 students on a voluntary basis responded to three mathematical items designed to complement the survey beliefs scales. The sample in this preliminary study is a convenience sample and is not representative of the population of the first-year students of the

primary school and pre-school teacher education in the country.

The survey beliefs scales

The survey beliefs scales were taken from the International Association for the Evaluation of Educational Achievement(IEA) "Teacher Education and Development Study in Mathematics(TEDS-M)" – "the first cross-national study to provide data on the knowledge that future primary and lower-secondary school teachers acquire during their mathematics teacher education" – including their beliefs (Tatto, et al., 2012, p. 18; see also Blömeke, 2012; Ingvarson, Schville, Tatto, Rowley, Peck, & Senk, 2013; Tatto, et al., 2008). The Likert-type scales cover three aspects of teachers' mathematics related beliefs:

    I. Beliefs about the nature of mathematics;

    II. Beliefs about learning mathematics;

    III. Beliefs about mathematics achievement.

The entire list of statements included in each of the scales is given in the Results section (Table 2). These statements represent two views, not equivalent with, yet related to:

– Conceptual and cognitive-constructivist orientations;

– Calculational and direct transmission orientations.

As described by Phillip (2007, p.303-304), "Actions of a teacher with a conceptual orientation are driven by an image of a system of ideas and ways of thinking she intends her students to develop; an image of how these ideas and ways of thinking can be developed; ideas about features of materials, activities and expositions and the students' engagement with them that can orient students' attention in productive ways; and an expectation and insistence that students will be intellectually engaged in tasks and activities. Although a teacher with a calculational orientation may share the general view that solving problems is important, the actions of such a teacher are driven by a fundamental image of mathematics as the application of calculations and procedures for deriving numerical results. Associated with a calculational orientation is a tendency to speak exclusively in the language of number and numerical operations, a predisposition to cast problem solving as producing a numerical solution, and a tendency to disregard context and to calculate upon any occasion to do so."

The mathematics items

The sample of mathematics items is restricted to only three due to the length of the beliefs survey and the perception of the author that the respondents would not volunteer to respond to a larger number of mathematics items, which was proved to be an accurate perception.

Item 1 – *Fraction multiplication representations*: Respondents were asked to analyze four possible pictorial representations of multiplication of two fractions (3 were correct models, two of which very similar) and choose if only one of them (which one), more than one (which ones), or none correctly modeled the operation.

This item is a modified version of the *Specialized content knowledge* sample item from the *Learning mathematics for teaching* database of *Mathematical knowledge for teaching* multiple-choice items (Ball, Thames, & Phelps, 2008). In the original version the respondents were asked to mark only one model which cannot be used to represent the fraction multiplication.

Item 2 – *Quadratic equation*: Respondents were asked to find the roots to a quadratic equation $ax^2 + bx + c = 0$ (for example, $x^2 - 7x + 12 = 0$) by using one of two methods:

- Finding the roots x_1 and x_2 as two whole numbers whose sum $x_1 + x_2 = -b$ is the negative of the linear coefficient b (the number 7, in the given example) and whose product $x_1 \cdot x_2 = c$ is the free term c (the

number 12, in the example) since the equation, for $a = 1$, is equivalent to the equation $x^2 - (x_1 + x_2) \cdot x + x_1 \cdot x_2 = 0$, i.e. to $(x - x_1)(x - x_2) = 0$ (the equation in the example is equivalent to the equation $x^2 - (3 + 4) \cdot x + 3 \cdot 4 = 0$, i.e. to the equation $(x - 3)(x - 4) = 0$).

- Finding the roots using the Quadratic Formula

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

A worked example was shown on top of the same page using each of the methods.

Item 3 – *The area of a triangle*: Respondents were asked to choose the method most likely to be used by grade 5 pupils[3] in finding the area of a triangle with vertices on 3 different sides of a grid composed of unit squares (see the Appendix) among the following ones:

A. Directly using the formula for the area of a triangle $A = \dfrac{u \cdot x_a}{2}$;

B. Using Pythagoras's Theorem $a^2 + \textit{б}^2 = u^2$ to calculate the lengths of the sides a, b, c of the gray triangle, then using Heron's Formula for the area of a triangle $A = \sqrt{C \cdot (C - a) \cdot (C - \textit{б}) \cdot (C - u)}$, $C = \dfrac{a + \textit{б} + u}{2}$;

C. Using the Distance formula $g = \sqrt{(u_1 - u_2)^2 + (\breve{u}_1 - \breve{u}_2)^2}$ to calculate the lengths of the sides a, b, c of the gray triangle, then using Heron's Formula for the area of a triangle $A = \sqrt{C \cdot (C - a) \cdot (C - \textit{б}) \cdot (C - u)}$, $C = \dfrac{a + \textit{б} + u}{2}$;

D. Finding the areas of the three rectangular triangles outside the gray triangle by halving the number of unit squares in the three corresponding rectangles, then subtracting the sum of these areas from the area of the whole grid;

E. Other _____

_____

This item was inspired by TEDS-M multiple-choice MCK item (Tatto, et al., 2007).

Results

The aggregated results of the survey are exhibited in Table 1 in the percentage of the responses endorsing the various scale statements. Following the course taken by the TEDS-M researchers (Tatto, et al., 2012), the responses "agree" or "strongly agree" are considered to represent unqualified endorsement, while the responses "strongly disagree", "disagree", "slightly disagree" or "slightly agree" as a non-endorsement or a weak endorsement of the statements.

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3 Since 2007, primary education in the country spans nine grades, grade 5 being analogous to grade 4 in the previous eight-year-long basic education. Primary school teachers teach all school subjects in grades 1-5, except foreign languages (English).

Table 1. Beliefs about mathematics and mathematics learning: percentage of endorsed statements

Beliefs about		Number of valid responses N	Agree-Strongly Agree %
The Nature of Mathematics	Mathematics as a Process of Inquiry	97	74.2
	Mathematics as a Set of Rules and Procedures	98	77.6
Learning Mathematics	Learning Mathematics through Active Involvement	101	75.2
	Learning Mathematics through Teacher Direction	93	21.5
Mathematics Achievement	Mathematics as a Fixed Ability	95	8.5

A strong support for the statements expressing beliefs consistent with the conceptual orientation (*Mathematics as a process of inquiry*) and the cognitive-constructivist orientation (*Learning mathematics through active involvement*) emerges from the responses since approximately 3 in 4 respondents agreed or strongly agreed with them. At the same time, 3 in 4 respondents also endorsed the calculational view of mathematics (*Mathematics as a set of rules and procedures*). The direct-transmission orientation (*Learning mathematics through teacher direction*) received support by only 1 in 5 respondents. Less than 1 in 10 respondents expressed strong endorsement of views of *Mathematics as a fixed ability*. A detailed presentation of each statement endorsement within each scale provides a more thorough picture (Table 2).

Table 2. Beliefs about mathematics and mathematics learning: percentage of various degrees of endorsement

Statements reflecting	Number of valid responses	Disagree-Strongly disagree	Slightly disagree-Slightly agree	Agree-Strongly Agree
	N	%	%	%
Mathematics as a Process of Inquiry:				
Mathematics involves creativity and new ideas.	101	5.0	50.5	44.6
In mathematics many things can be discovered and tried out by oneself.	99	6.1	37.4	56.6
If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts).	102	9.8	32.4	57.8
Mathematical problems can be solved correctly in many ways.	102	2.0	17.6	80.4
Many aspects of mathematics have practical relevance.	101	2.0	23.8	74.3
Mathematics helps solve everyday problems and tasks.	102	7.8	36.3	55.9
Mathematics as a Set of Rules and Procedures:				
Mathematics is a collection of rules and procedures that prescribe how to solve a problem.	100	7.0	35.0	58.0
Mathematics involves the remembering & application of definitions, formulas, mathematical facts, & procedures.	101	3.0	13.9	83.2
When solving mathematical tasks, you need to know the correct procedure, else you would be lost.	101	15.8	19.8	64.4
Fundamental to mathematics is its logical rigor and precision.	102	4.9	34.3	60.8
To do mathematics requires much practice, correct application of routines, and problem solving strategies.	102	2.9	21.6	75.5
Mathematics means learning, remembering, and applying.	101	0.0	15.8	84.2
Learning Mathematics through Active Involvement:				
In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.	102	0.0	9.8	90.2
Teachers should allow pupils to figure out their own ways to solve mathematical problems.	101	13.9	39.6	46.5
Time used to investigate why a solution to a mathematical problem works is time well spent.	101	0.0	11.9	88.1

Pupils can figure out a way to solve mathematical problems without a teacher's help.	102	23.5	42.2	34.3
Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient.	101	17.8	31.7	50.5
It is helpful for pupils to discuss different ways to solve particular problems.	102	0.0	11.8	88.2
Learning Mathematics through Teacher Direction:				
The best way to do well in mathematics is to memorize all the formulas.	101	15.8	40.6	43.6
Pupils need to be taught exact procedures for solving mathematical problems.	101	2.0	23.8	74.3
It doesn't really matter if you understand a mathematical problem, if you can get the right answer.	102	50.0	31.4	18.6
To be good in mathematics you must be able to solve problems quickly.	101	42.6	33.7	23.8
Pupils learn mathematics best by paying attention to the teachers' explanations.	102	1.0	10.8	88.2
When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed.	102	50.0	34.3	15.7
Non-standard procedures should be discouraged because they can interfere with learning the correct procedure.	96	13.5	44.8	41.7
Hands-on mathematics experiences aren't worth the time and expense.	100	35.0	40.0	25.0
Mathematics as a Fixed Ability:				
Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary.	96	29.2	46.5	24.0
To be good at mathematics, you need to have a kind of "mathematical mind".	102	38.3	42.2	19.6
Mathematics is a subject in which natural ability matters a lot more than effort.	100	36.0	38.0	26.0
Only the more able pupils can participate in multi-step problem-solving activities.	101	41.6	37.6	20.8
In general, boys tend to be naturally better at mathematics than girls.	102	67.6	18.6	13.7
Mathematical ability is something that remains relatively fixed throughout a person's life.	101	10.9	33.7	55.4
Some people are good at mathematics and some aren't.	102	18.6	32.4	49.0
Some ethnic groups are better at mathematics than others.	102	61.8	23.5	14.7

Among the statements related to *Mathematics as a process of inquiry*, the statements which received strongest support were "Mathematical problems can be solved correctly in many ways" (by slightly above 4 in 5 respondents), and "Many aspects of mathematics have practical relevance" (by almost 3 in 4 respondents). The percentage of the respondents expressing slight disagreement or slight agreement with "Mathematics involves creativity and new ideas" (half of the sample) is somewhat higher than the number of the respondents fully endorsing it (45 %).

Strongest endorsement among statements referring to *Mathematics as a set of rules and procedures* was granted to the statements "Mathematics means

learning, remembering, and applying", and "Mathematics involves the remembering and application of definitions, formulas, mathematical facts, and procedures" (by more than 8 in 10 respondents), as well as to "To do mathematics requires much practice, correct application of routines, and problem solving strategies" (by more than 3 in 4 respondents). Each of the statements on this scale (calculational orientation) was fully endorsed by more than half of the respondents.

Some of the statements most consistent with *Learning mathematics through active involvement* received the highest support in comparison to statements on the other subscales: "In addition to getting a right answer in mathematics, it is important to understand why the answer is correct" (by more than 9 in 10 respondents endorsed it), "It is helpful for pupils to discuss different ways to solve particular problems", and "Time used to investigate why a solution to a mathematical problem works is time well spent" (both by almost 9 in 10 respondents). Among the statements reflecting the cognitive-constructionist orientation the least support was granted to the statement "Pupils can figure out a way to solve mathematical problems without a teacher's help" (approximately 1 in 3 respondents endorsed it fully, and 1 in 4 respondents rejected it).

The responses to the statements on the scale of *Learning mathematics through teacher direction* differed to various degrees depending on the statement. The strongest support was expressed for "Pupils learn mathematics best by paying attention to the teacher's explanations" (by almost 9 in 10 respondents), and for "Pupils need to be taught exact procedures for solving mathematical problems" (by almost 3 in 4 respondents). The weakest endorsement for a statement consistent with the learning mathematics through direct transmission orientation was afforded to "When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed" (by less than 1 in 6 respondents slightly agreeing or slightly disagreeing, and by half of the re-

spondents fully rejecting it), and to "It doesn't really matter if you understand a mathematical problem, if you can get the right answer" (by less than 1 in 5 respondents endorsing it, and half of the respondents rejecting it).

Only two of the statements included in *Mathematics as a fixed ability* scale received support by approximately a half of the respondents, "Mathematical ability is something that remains relatively fixed throughout a person's life", and "Some people are good at mathematics and some aren't". In contrast, the statement "In general, boys tend to be naturally better at mathematics than girls" was fully rejected by more than two thirds of the respondents. Similarly, approximately 6 in 10 respondents strongly disagreed or disagreed with the statement "Some ethnic groups are better at mathematics than others".

Before any deeper analysis is to be undertaken, several questions immediately arise: What is the nature of these beliefs? Which beliefs are teachers-students inclined to act upon? Since the survey was conducted with the first-year students, it was not possible to gather observational data of their instructional decisions and their teaching practice, reserved for the final two years of their university studies. The attempt to address the above mentioned questions might benefit from a review of the responses on the mathematics items (Table 3).

On Item 1 (Fraction Multiplication Representations), more than a half of the respondents chose only one representation as being the only one which appropriately models the fraction multiplication, although only one of the given four representations was incorrect. Since two of the given four representations were almost identical, the question of how some of the respondents eliminated one of them seems quite legitimate.

To find the root of the given quadratic equation in Item 2, more than 8 in 10 respondents chose to use the Quadratic formula, although the other method is far more intuitive, makes sense in itself, and is less cumbersome than applying the formula.

Table 3. Responses to the mathematics items

		Frequency	Valid %
Valid	Item 1 – *Fraction multiplication representations*		
	There is only one correct representation (the incorrect representation chosen)	3	4.3
	There is only one correct representation (a correct representation chosen)	34	48.6
	There are more than one correct representations (the incorrect representationalso chosen)	6	8.6
	There are more than one correct representations (the correct representations chosen)	14	20.0
	There are no correct representations	13	18.6
Total		70	100
Missing		1	
Valid	Item 2 – *Quadratic equation*		
	The Quadratic Formula used	58	85.3
	Logical solution	10	14.7
Total		68	100
Missing		3	
Valid	Item 2 – *The area of a triangle (Choose grade 5 pupils' solution strategy!)*		
	Standard formula	24	33.8
	Pythagoras's formula & Heron's formula	22	31.0
	Distance formula & Heron's formula	6	8.5
	Count squares, take halves of areas of rectangles, add them, subtract the sum	15	21.1
	Other (incorrect) solutions	4	5.6
Total		71	100
Missing		0	

In choosing the strategy most likely to be used by grade 5 pupils in finding the area of the given triangle, only 1 in 5 respondents made a reasonable decision having in mind the non-applicability or the complexity (the required academic maturity) of the solving strategies offered, and the grade level of the pupils expected to solve the problem.

Discussion

The research has many limitations, including the following: the sample of the respondents is not representative; the number of mathematics items included in the survey is very small; the use of the results from the Likert-type scales has well-known limitations (Phillip, 2007), especially when such instruments are used for measuring the beliefs isolated from the knowledge or actual instructional practices. The results cannot be generalized, yet they open up a key-hole view of the landscape of beliefs of the teacher education undergraduates in the country.

The results from the beliefs survey are not surprising. The self-professed beliefs of the first-year students (future primary school and pre-school teachers) in the country are consistent with the beliefs expressed by the majority of students in their final year of teacher education studies in most of the countries in the TEDS-M (Tatto, et al., 2012). It would be interesting to compare the results of the first-year students survey with the results of the survey conducted with the fourth-year students at the end of their teacher education university studies in the larger national study. As Philipp (2007) elaborated in his comprehensive review of the research on the mathematics teachers' beliefs and affects, belief systems are resistant to change and there is no conclusive scientific evidence that teacher education provides a successful scaffolding for a permanent change in the pre-existing beliefs of prospective teachers.

The pattern of beliefs expressed by the respondents in the study strongly endorses the view of mathematics as a process of enquiry – a pattern which appears in the responses of the future prima-

ry teachers in all but one country in TEDS-M (Tatto, et al., 2012). An overwhelming majority of the first-year students of teacher education in our study fully acknowledged that there was a wide spectrum of solution strategies when attempting to solve mathematical problems and that mathematics had practical relevance. A majority of them also endorsed the beliefs that engagement in mathematical tasks lead to the discovery of new concepts, patterns and connections, although approximately one in three of the respondents expressed slight reservations. The same is the case with the beliefs referring to the possibility of discovering many things in mathematics by oneself and the possibility of solving everyday problems by means of mathematics. Although only a very small percentage of the respondents disagreed with the view of mathematics as a creative human activity which involves new ideas, a half of the respondents had some reservations regarding it as such. These findings necessitate further inquiry into the nature of these reservations in order to be able to design the appropriate *inquiry based learning* mathematical activities within teacher education courses in which future teachers will experience the joy of discovering and creating mathematical knowledge for themselves and by themselves.

At the same time, a great majority of the respondents in this study fully endorsed the calculational view of mathematics, i.e. mathematics as a set of rules and procedures, strongly endorsing beliefs that mathematics involves the remembering and application of definitions, formulas, mathematical facts, and procedures, and that doing mathematics requires much practice, correct application of routines, and problem solving strategies. Again, this pattern was common across the countries in TEDS-M (Tatto, et al., 2012), with few exceptions, and with a considerable diversity in the extent to which future teachers express support for the corresponding statements. The strong endorsement of this kind of beliefs can be seen as a consequence of the established tradition of mathematics instruction in primary and secondary

schools in the country, and in other countries world-wide.

The cognitive-constructionist orientations, i.e. the view of learning mathematics through active involvement, received an overwhelming support from the respondents in the study, again in sync with the results from TEDS-M (Tatto, et al., 2012). Future teachers recognize the importance of investigating the solutions to a mathematical problem and the need for a justification of the answer, as well as how significant it is for pupils to discuss different solution strategies. The belief that pupils can find the ways to solve mathematical problems without their teacher's help received the least support among the beliefs on this scale. These findings could be explained by the absence of opportunities to observe pupils' mathematical thinking available to prospective teachers at the start of their teacher education. A note should be taken by teacher educators in terms of providing such opportunities within mathematics methodology courses.

The direct transmission view of mathematics learning was rejected by the majority of the prospective teachers in the sample – a finding consistent with the findings in TEDS-M (Tatto, et al., 2012). The statements reflecting beliefs about learning mathematics through teacher instruction were endorsed to a various degree depending on the statement, with strongest support for learning mathematics by attending to the teacher's explanations and for the need for teachers to teach pupils the exact procedures for solving mathematical problems. The majority of the respondents fully rejected the belief that more emphasis should be put on getting the correct answer than on the process of reasoning, and that getting the right answer trumps understanding the mathematical problem. This rejection is compatible with the above-discussed endorsement of the beliefs on learning mathematics through active involvement.

Similarly to the findings in TEDS-M (Tatto, et al., 2012), mathematics as a fixed ability is the view which was not endorsed by most of the respondents in the study. A great majority of the future primary

and pre-school teachers in the sample had strongest objections to gender and ethnic bias regarding achievement in mathematics. Still, almost half of them agreed that mathematical ability was something that remained relatively constant throughout a person's life and that some individuals are naturally better at mathematics than others. Less than a half of the respondents also had slight hesitations in agreeing or disagreeing with the belief that having a "mathematical mind" was a prerequisite for the success in doing mathematics, as well with the belief that a natural ability matters a lot more than effort; a quarter of the respondents endorsed these beliefs. Supporting the view of mathematics as a fixed ability carries a danger for teachers when designing and implementing challenging instructional activities to address the needs of only a selected few pupils perceived as being good in mathematics, and not properly supporting the development of mathematical thinking of every child in their classroom.

The results on the mathematical items used in the study are quite surprising. Although the vast majority of the respondents unequivocally supported the belief that mathematical problems can be solved in many ways, when asked to choose whether one or more pictorial representations accurately model a given example of the fraction multiplication, more than a half of them focused on finding only one representation, although three of the four representations were correct and two of them were almost indistinguishable. It is possible that primary and secondary mathematics education succeeded in enabling students to recognize and profess the desirable "mantra of the day" when it comes to learning and doing mathematics, yet failed to equip them with the knowledge required to act in consistence with what they so readily acknowledge.

Since the vast majority of the respondents fully endorsed the statements which describe mathematics as involving remembering and correct application of formulas, mathematical facts, and routine procedures, the choice to use the Quadratic formula over a much simpler method for finding the roots of a quad-

ratic equation does not seem surprising. Not surprising, yet disturbing! This established mode of doing mathematics by universally applying formulas whenever possible, or not possible, and when applying simple logic would be much more productive, has to be brought to the attention of future teachers. The design of mathematical activities in which solving problems using reason instead of senseless application of formal procedures, as well as enabling future teachers to carefully select when it is most appropriate to apply formulas, has to be one of the primary tasks of teacher education mathematics courses, and as such it has to be explicitly defined as an educational goal.

This last argument also refers to the results obtained on the last mathematics item when first-year students of teacher education were asked to make an educated guess regarding the choice of a strategy an 11-year old pupil would be expected to use when finding the area of a triangle. An overwhelming majority of the respondents could not resist the urge to choose a formula, any formula, even when it was impossible to use it, or even when it required certain higher level of mathematics knowledge, not accessible to an average pupil in primary grades. It can be argued that the knowledge of primary mathematics curriculum is to be acquired by future teachers by the end of the teacher education studies. Yet, the fact remains that for the time being, the university-level mathematics education of future teachers has to counteract the negative consequences of bad education in primary and secondary grades, namely the deeply rooted habits of the mind to disregard common sense and logic in favor of an unselective use of formal procedures.

In order to observe these findings from a proper perspective, it is informative to look at the findings in TEDS-M. Teachers' mathematics content knowledge (MCK) and teachers' mathematics pedagogy content knowledge (MPCK) are positively related to teachers' conceptual and cognitive-constructionist orientation, and negatively related to teachers' calculational and direct transmission orientation, i.e. "... within countries there was a general tendency for future teachers who endorsed the beliefs that mathe-

matics is a process of inquiry and that learning mathematics requires active involvement to have relatively greater knowledge of mathematics content and pedagogy than those who rejected those beliefs. Similarly, there was a general tendency within countries for those future teachers endorsing the beliefs that mathematics is a set of rules and procedures, learning mathematics requires following teacher direction, and mathematics is a fixed ability to have relatively lesser knowledge of mathematics content and pedagogy than those who rejected those beliefs" (Tatto, et al., 2012, p.169). This issue calls for further research on teachers' beliefs to be conducted inseparably from the research on teachers' knowledge of mathematics content and of mathematics pedagogy, as well as on teachers' instructional practices.

If beliefs serve as lenses which filter how an individual sees the world, how can they be changed? As noted by Thompson (1992, as cited by Phillip, 2007, p.260-261) "teachers often assimilate new ideas to fit their existing schemata instead of accommodating their existing schemata to internalize new ideas". Many researchers attempted to develop the mechanisms for influencing practicing and prospective teachers' beliefs, and among the ones which produced some success are the following (Phillip, 2007). Teachers' beliefs change if teachers evidence positive changes in student learning outcomes; providing (prospective) teachers with opportunities to learn about students' mathematical thinking and reflect upon these experiences which successfully influence their beliefs, as well as immersing (prospective) teachers in a community so that they become enculturated with beliefs through cultural transmissions.

Changing teacher education in line with these beliefs has to happen if positive change is to take place, but before it can happen, it is essential to reflect upon one more finding from TEDS-M: The pattern of beliefs held by future teachers matches the pattern of beliefs of teacher educators (Tatto, et al., 2012). Simply introducing new, reformed mathematics (and mathematics methodology) curricula is not enough. Teachers' beliefs and knowledge develop as a result of their learning experiences since their early age ("*years of apprenticeship*"), and the process of mathematics learning (and of becoming a teacher of mathematics) is a process of "*enculturation*", of becoming a member of a community of learners of mathematics.

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Appendix

III. Врсниците на учениците од петто одделение деветгодишно основно образование во дваесетина држави добиле задача слична на следнава задача.

Површината на секое од квадратчињата во мрежата е 1 cm². Пресметајте ја површината на сивиот триаголник.
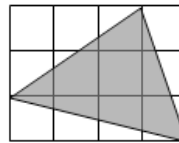
Според Вас, на кој начин се очекува од учениците да ја решаваат задачата?

Одберете ЕДЕН од понудените приоди за решавање, за кој верувате дека се очекува да го применат учениците, и заокружете ја буквата пред него.

А. Со формулата за површина на триаголник, $P = \dfrac{a \cdot h_a}{2}$.

Б. Со пресметување на хипотенузите на правоаголните триаголници со помош на Питагоровата теорема, $a^2 + b^2 = c^2$, и со примена на Хероновата формула за површина на триаголник, $P = \sqrt{S \cdot (S-a) \cdot (S-b) \cdot (S-c)}$, каде што $S = \dfrac{a+b+c}{2}$.

В. Со помош на формулата за растојание меѓу две точки со дадени координати, $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, и со Хероновата формула за површина на триаголник, $P = \sqrt{S \cdot (S-a) \cdot (S-b) \cdot (S-c)}$, $S = \dfrac{a+b+c}{2}$.

Г. Со пресметување на површината на правоаголната мрежа и со одземање на збирот на површините на правоаголните триаголници во мрежата сместени надвор од сивиот триаголник, а нивната површина се пресметува со преполовување на површините на соодветните правоаголници.

Д. На друг начин: _____
_____
_____

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УВЕРЕЊА БУДУЋИХ УЧИТЕЉА И ВАСПИТАЧА О ПРИРОДИ МАТЕМАТИКЕ И О УЧЕЊУ МАТЕМАТИКЕ

Резиме: Универзитетске студије, између осталог, имају за циљ да омогуће учитељима и васпитачима да подрже развој математичких компетенција ученика на раном узрасту. Иако је знање о математичким садржајима главна компонента стручног знања неопходног за предавање математике, стручна уверења наставника о томе шта је математика и како се она учи имају значајан посреднички ефекат на успех наставника у пружању стварних могућности ученицима да уче математику на смислен начин.

Истраживачки циљ студије је био да се анализирају уверења будућих наставника о природи математике и учењу математике. Проучавање уверења (и знања) наставника мотивисано је ставом да уверења (и знање) учитеља обликују њихов рад, као и да пракса наставника у учионици утиче на развој математичких знања ученика.

Истраживање објављено у раду део је прелиминарне студије урађене у склопу већег националног пројекта ("Уверења будућих учитеља и васпитача о природи језика и математике и о настави и учењу језика и математике") који финансира Универзитет Кирил и Методије у Скопљу. Узорком су обухваћена сто два студента прве године (осамдесет девет студенткиња, 87% узорка), са смера за учитеље (седамдесет један испитаник) и са смера за васпитаче (тридесет један испитаник). Упитник се састојао од скале уверења везаних за математику, учење математике и математичка постигнућа, развијене у оквиру студије ТЕДС-М ("Студија о образовању и развоју наставника у математици"). Изјаве представљају два става која нису еквивалентна, али су повезана са: концептуалним и когнитивно-конструктивистичким оријентацијама, као и са калкулационим концептом и оријентацијом ка директном преношењу знања. Анкета је спроведена у првој недељи семестра, када се студенти први пут сусрећу са наставом математике на универзитету. Подузорак од седамдесет једног студента одговарао је, на добровољној основи, на три питања у вези са математиком која су креирана да допуне ставове исказане на скали уверења. Из одговора се види да постоји снажна подршка изјавама које изражавају уверења у складу са концептуалном оријентацијом (математика као процес истраживања) и когнитивно-конструктивистичком оријентацијом (учење математике кроз активно укључивање); око 75% испитаника се сложило или потпуно сложило с њима. Истовремено, 75% испитаника је такође подржало калкулациони концепт математике (математика као скуп правила и процедура). Оријентација ка директном преношењу знања (учење математике путем инструкција добијених од наставника) добила је подршку само једног од пет испитаника. Тек сваки десети испитаник изразио је снажну подршку ставу о "Математици као фиксној способности".

Резултати су указали на разлику између самопрокламованих уверења студената и приступа које су користили у решавању математичких задатака. Иако је велика већина

испитаника недвосмислено подржала уверење да се математички проблеми могу решити на више начина, када се од њих тражило да одговоре да ли једна или више сликовних представа тачно моделирају дат пример множења разломака, више од половине испитаника фокусирало се на проналажење само једне визуелне представе, иако су три од четири слике биле тачне, а између двеју готово да није било разлике. Одговори на други математички задатак указали су на ув   режени начин решавања математичких задатака применом формула кад год је то могуће или није могуће, и то онда када би примена просте логике дала много боље резултате. Оспособљавање будућих наставника да умеју пажљиво да одаберу када је најпогодније применити формуле тако што ће креирати математичке активности у којима ће се решавати проблеми уз помоћ здравог разума, а не бесмисленом применом формалних процедура, мора да буде један од примарних задатака наставе математике на факултетима који образују будуће учитеље и васпитаче. Овај аргумент односи се и на резултате добијене у вези са последњим математичким задатком, када је од испитаника тражено да идентификују стратегију коју би једанаестогодишњи ученик користио да одреди размере троугла. Велика већина испитаника је одабрала формулу, било коју формулу, чак и када је то било немогуће или чак када би то захтевало виши ниво математичког знања, који није доступан просечном ученику основне школе. Чини се да универзитетско образовање будућих наставника мора да се позабави дубоко укорењеном менталном навиком да се занемарују здрав разум и логика у корист неселективне употребе формалних математичких процедура.

Налази истраживања указују на потребу за пружањем специфичних могућности за учење у иницијалном образовању наставника како би се будућим наставницима помогло да развију кохерентна математичка знања и конзистентна професионална уверења. Увођење нових, реформисаних математичких курикулума (и математичких метода) није довољно. Уверења и знања наставника развијају се као резултат њихових искустава стечених учењем од раног узраста („године припремног рада“), а процес учења математике (и стицања звања наставника математике) јесте процес „културизације“, током којег се постаје део заједнице људи који уче математику.

Кључне речи: математичко образовање, учитељи, васпитачи, стручно знање, уверења.