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Milica D. Antić
"Branko Ćopić" Primary School, Belgrade, Serbia

Olivera J. Đokić[1]
University of Belgrade, Teacher Education Faculty, Serbia

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The Development of the Components of the Linear Measurement Concept among the Pupils of the First Grade of Primary School[2]

Extended summary

The results of international research projects, such as TIMSS, indicate that pupils' understanding of geometry content often lags behind their understanding of the content of other areas of mathematics. To discover the cause of this phenomenon, we focused on the initial mathematics education and searched for the main reasons for pupils' underachievement in this field. We analysed the first-graders' achievement in the field of Measurement and Measures, focusing primarily on the linear measurement. The goal of the research was to determine how successfully the pupils had mastered the linear measurement concept consisting of components on which the measurement procedure is based. The components include: partitioning, unit iteration, transitivity, conservation, accumulation of distance, and relation to number.

The key conclusions of the study are presented further in the paper.

The researchers used descriptive method. The research was conducted on a sample of 47 first-graders of "Branko Ćopić"primary school in Belgrade, while interview was a selected research technique. The researchers pre-planned the conversations with the pupils and prepared the necessary materials.

1. *Partitioning*. Our first task involved examining pupils' understanding of the concept which implies that the length of an object represents the length partitioned into the units of meas-

1 olivera.djokic@uf.bg.ac.rs

2 The paper represents a modified and amended master thesis entitled *Introducing the concept of linear measurement in the initial mathematics teaching*, defended at the Teacher Education Faculty, University of Belgrade (mentor Olivera J. Đokić, PhD).

urement of the same length. The pupils were supposed to choose the units of measurement (of the same length or of different lengths), and then apply them to show the manner in which they use them in the process of measurement. As far as the first task is concerned, more than a half of the pupils, 26 in total (55.32%), used the units of different lengths for the linear measurement of objects, which means that they failed to adopt the knowledge of this component. Less than a half of the pupils, 21 in total (44.68%), used the units of the same length. This result indicates that pupils do not understand the partitioning component.

2. *Unit iteration.* This component involves the ability to think about a unit of measurement as a part that constantly iterates along the whole length of an object which is being measured – without any overlap or leaving blank spaces. Our second task consisted of two activities aimed at determining pupils' adoption of this component. In the first activity the pupils were asked to comment on the unit iteration procedure demonstrated by the examiner, while in the second activity they had to carry out the unit iteration procedure and measure the length of an object by themselves. When asked to comment on the manner in which the examiner had conducted the procedure, 28 pupils (59.57%) observed that the demonstrated unit iteration procedure was not correct, while 19 pupils (40.43%) could not evaluate the examiner's procedure appropriately. As for the second activity, 20 pupils (42.56%) carried out the procedure correctly, while as many as 27 pupils (57.44%) failed to do so. Eight pupils (17% or nearly one-fifth of the total number of pupils) who identified the correct procedure when it was conducted by the examiner could not repeat it when they were supposed to do it by themselves, which indicates that the understanding of the correct measurement procedure is still not sufficient.

3. *Accumulation of distance*. The purpose of the third task was to determine whether the pupils understand that the length of an object, in the process of unit iteration, is the distance from the beginning of the first, to the end of the last unit of measurement. Many pupils, 30 in total (63.83%), perceive the measuring number as the total number of linked straws. Furthermore, when asked to provide their answer to the question, these pupils pointed to a red straw whose length was being measured, from the beginning of the straw, to its end. The pupils who answered incorrectly (17 pupils, 36.17%) explained that the obtained number is in the interval of the last unit of measurement, i.e. that the measuring number is the number of the straw that was added last.

4. *Transitivity*. The fourth task consisted of two activities. The pupils first had to assort the straws in the order from the longest straw to the shortest one. All 47 pupils (100%) performed the first activity correctly. In the second activity, they had to compare the given straw with the other two straws. Only four pupils (8.51%) failed to do this activity, whereas 43 pupils (91.49%) completed it successfully. The pupils who did the task incorrectly claimed that the straw could not be longer than the other one and shorter than the third straw at the same time. They concluded that the straw could be either shorter or longer.

5. *Conservation.* Pupils' understanding of this component of the linear measurement concept was checked in the fifth task. According to the results, as many as 36 pupils (76.60%) think that the length of a straw changes when the straw is moved from one place to another, while 11 pupils (23.40%) understand the component fully. Given that conservation is a very important component which, if not learnt and adopted fully, renders any valid measurement impossible, the number of pupils who have not mastered this component gives rise to concern.

6. *Relation to number*. The sixth task tested the pupils' adoption of the relation to number component. The pupils were shown two lines of equal length and both made up of straws. There were six blue straws in the first line and eight red straws in the second line. The length of each blue straw was shorter than the length of each red straw. The pupils had to count the units of measurement in both lines and then to compare the lines in terms of their length. Twenty-two pupils, which is less than a half of the total number of pupils (46.80%), performed the task correctly. They observed that the straws were shorter (longer) in the line in which their number was higher (lower). The 23 pupils (53.20%) who performed the task incorrectly justified their answers by saying that the higher the number of straws in one line, the longer that line would be. Their explanation only confirmed that they did not fully adopt the relation to number component.

The research results indicate that there is a huge discrepancy between the pupils' adoption of the linear measurement concept on one hand, and its components, on the other. It is also indicative that the mathematics curriculum which was in force at the time of the research, and which had a great impact on both primary school teachers' work and textbook writers, did not provide a solid foundation and support to primary school teachers. The explanation for the pupils' low level of understanding of the linear measurement concept can be found in the small number of lessons allotted to this topic in the curriculum, as well as in the insistence on using the metre and performing the measurements by using tools on one hand, and neglecting the key components of the linear measurement concept in the process of measurement, on the other. If the results show that the first-graders have not developed the specific components of the concept on which the measurement procedure is based (e.g. partitioning and conservation), there is no point in using rulers for measurement and insisting on carrying out measurements by using tools. The new First Grade Curriculum partly contains the modifications that are in line with the results of our research. The abandonment of the metre as an instrument and performing measurement by using the standard units of measurement are significant modifications in the field that was also the topic of our research. The new curriculum stipulates an educational outcome that pupils, after finishing the first grade of primary school, must be able to measure the length by using the non-standard unit of measurement. The instructions also contain a suggestion that the same, or different, units of measurement can be used for measuring the same object, and that measurement should be performed by unit iteration. In addition, the curriculum recommends that measurement results should be presented in the form of tables/diagrams. However, we believe that *all six components of the linear measurement concept* should be included and introduced gradually in teaching geometry.

Our suggestion for further research is that linear measurement concept should be taught in terms of all its key components, by means of practical incentives and pupils' spatial experience, as well as by providing examples which would help pupils to perceive geometrical objects in space and their relationships, compare the sizes of the objects, and other similar activities.

The ways and order of forming the linear measurement concept with all its components which precede the introduction of the standard linear measurement unit are the open-ended questions that require further research.

Keywords: linear measurement, key concepts in linear measurement, teaching and learning geometry, mathematics curriculum.

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