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Algorithmic and Conceptual Approach to Figure Surface Measurement[2]

Extended summary

Surface measurement is an important topic of the school mathematics curriculum which is closely related to the application of mathematics in the real world. Students' achievements in mathematics demonstrate that they score low in the area of surface measurement (Martin and Strutchens, 2000; Lin and Tsai, 2003; Jelić i Đokić, 2017). Research suggests that the inflexibility students demonstrate in tasks dealing with surface measurement may be the result of the mathematics curriculum (Strutchens et al., 2001). Two factors that negatively affect students' understanding have been singled out: 1) an algorithmic/numerical approach to making calcuations is dominant in teaching practice (Tan,1995; Tan 1999; Huang and Witz, 2013), and 2) a lack of connection between 2D geometry and surface measurements in the curriculum (Stephan and Clements, 2003; Kordaki and Balomenou, 2006). Stephan and Clements (2003) state that students most often learn to measure a rectangle's surface by multiplying the measured values of their sides. This instruction is confusing as a 2D surface cannot be understood by simply multiplying the measured values of its length and width. Students' lack of understanding as to how the surface of triangles or parallelograms can be observed in relation to the surface of a rectangle also demonstrates how surface comprehension is limited to the application of formulas (Tan, 1999). The results of numerous studies suggest that there is a big "cognitive leap" from counting the squares that tile a figure to the conceptual understanding of the formulas (Van de

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Walle, 2007). Understanding the concept of surfaces requires learning and coordinating several ideas and procedures, which is a cognitive challenge for students (Battista, 2004; Clements and Stephan, 2004; Battista, 2007). The basic concepts for understanding surfaces include the tiling of figures without gaps or overlaping units, dividing surfaces into equal units of measurement, counting the units of measurement, considering combined units of measurement, understanding the structure of rows and columns, and connecting a number of squares by length and width (Battista, 2007; Sarama and Clements, 2009). Some researchers tried to identify the levels and characteristics of the phases students go through when developing and understanding geometric concepts and the concept of measurement. Describing the characteristics of the hierarchy of those levels is a valuable tool for teachers and researchers and it can be used when researching students' understanding ((Kow and Yeo, 2008; Usiskin, 2012; Huang and Witz, 2013; Herendine-Konya, 2015).

The aim of this research paper is to examine the level and quality of knowledge about surface measurements among pupils of the fourth grade of primary school. As basic indicators of pupils' knowledge we used the parameters defined by (Huang and Witz, 2013): understanding the concept of a surface, the ability to explain the meaning of a formula, the ability to differentiate between a circumference and a surface, strategies for studying surfaces, the ability to identify an incorrect solution. The descriptive method and testing technique were used in the research. The research uses modified tasks used in the previous research (Huang and Witz, 2013). As the understanding of surfaces and differentiating between a circumference and a surface were examined through interviews, we decided to use tests, and formulated our tasks based on transcripts of the interviews. Our sample consisted of two classes (45 students) of the fourth grade of a primary school in Belgrade. Given that this research required teacher cooperation and consent, the sample is appropriate. Fourth grade students were chosen because the surfaces of geometric figures and bodies are covered in that school year. The research was conducted in May, when all content related to surface measurement had been covered.

The results showed that (64.4%) of the students understand the idea of tiling (covering) a figure and comparing this to a given unit of measurement. The research used a task where the given figure was shown on a square grid, so there was no need for participants to divide the figure, but instead to simply notice the units of measurement, which is to say that the tasks were basic. A high success rate was expected on this task because of this fact. Falling success rates on other tasks further show that the previously listed skills are basic ideas necessary for students to develop a deeper understanding of surface measuring. Only 11.1% of students demonstrated a certain degree of understanding the meaning of the formulas, while 26.7% of students knows the difference between a circumference and a surface; the strategies for calculating the surface of a right triangle (half of the surface of a rectangle) were used successfully by 17.8% of the students when the triangle was on a square grid, and 15.6% when the triangle was not on a grid. Even though the difference in success rates when the figure is shown on a grid and when only its dimensions are provided is small, the results do show that students are intuitively more familiar with the idea of tiling than with an algorithmic aproach. In the end, only 8.9% of the students managed to correct a wrong solution to a given task. Since identifying and correcting the incorrect solutions represents the highest level of knowledge (Huang and Witz, 2013), and

in the context of the scores from the previous tasks, such resutls were expected. This does not mean that these types of tasks should not be used in class, but rather that students need aid in understanding the material being covered, and in reaching the highest level of knowledge possible.

Given the results shown, it is safe to conclude that the students in our sample were taught using an algorithmic aproach to surface measurement. Apart from their low scores, this is further backed-up by the strategies the students tried to use to reach their answers. The method most often used by the students was to apply certain formulas, the ones which were not appropriate. Even though all of the figures were 2D, the students tried to use the formulas to calculate the surface area of a cuboid, as well as the formulas used for measuring the circumference of certain shapes. Understanding surface measurement is stuck in a rigid frame of rules and formulas. It is our belief that the majority of the students who participated in our research were not able to make a big "cognitive leap" and use the idea of tiling figures using certain units of measurement to reach the conceptual understanding of the formulas and procedures in surface measurement. It should be noted that the tests were conducted at the end of the fourth year, when the units of measurement, as well as the surfaces of the squares, cubes and cuboids had already been covered in class. It is to be expected that students will completely understand the idea of a surface. The aim of our research is to put an emphasis on the need to allow enough time for students to understand basic units of surface measurement and to learn to calculate a surface using the conceptual approach of dividing and tiling a figure. This represents the basis of understanding the surfaces of other figures. Once students have a good grasp of and can visualise the strucutre of a surface (in relation to units of measurement), they will be able to connect the structure with a surfrace formula.

Keywords: measurement, surface area, algorithmic and conceptual approach.

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