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A problem-solving process using the Theory of Didactical Situations: 500 lockers problem[2]

Extended summary

Focusing on the active participation of individuals, Brousseau's Theory of Didactical Situations [TDS] (2002) states that "Doing mathematics does not consist only of receiving, learning and sending correct, relevant (appropriate) mathematical messages" (p. 15). The didactical situation is made up of five phases which can be summarized briefly as; (*i*) *devolution* phase where the teacher transfers the responsibility to the students, (*ii*) the *action* phase where the students come up with new hypotheses on how to solve the problem, (*iii*) the *formulation* phase where the students articulate their hypothesis (*iv*) the *validation* phase where the hypotheses are tested for their validity, and finally (*v*) the *institutionalization* where the teacher offers possible solutions to the given problem and presents the problem in different contexts where the earlier solutions are the basis for understanding (Brousseau, 2002).

The TDS constitutes the framework for this research since the students endeavor to acquire knowledge on their own and, most importantly, since exploring how students learn within the process, rather than how teachers teach the subject, is the baseline for the present research. In this context, this study aims to examine the mathematical thinking skills of the students in an adidactical situation through an inquiry-based problem solving. Therefore, the study is important in terms of providing a basis on how to conduct a didactical situation within

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TDS, shifting the locker problem in a different context and examining the students' behaviors in an environment which requires of them to get involved in higher thinking processes.

Method. The participants of this case study were 16 voluntary undergraduate students. The locker problem was investigated to find out the mathematical thinking processes of the pre-service teachers. The problem is about opening and closing the doors of the multiples of all locker numbers respectively; i.e. first student opens all the doors, the second one closes the doors with even numbers, the third one changes the state of every third locker. "*How many lockers will be open when all 500 students open or close the doors in the way described above?*" The researchers' notes, video camera recording, sketches of the groups on the delivered papers were used in the deductive analysis, in which the data were analyzed according to an existing framework (Patton, 2002, p.453). The data analysis was conducted according to the TDS concepts, i.e., the stages of devolution, action, formulation, validation and institutionalization.

Results and Discussion. Devolution Stage: The aforementioned problem was introduced to the students and the expectations from the groups were stated in order to have an effective problem-solving process. *Action Stage:* The most important indicator of this phase was that the students passionately discussed the possible solutions within the groups and put forth their strategies. *Formulation Stage*: The students who struggled for the solution through trial-and-error search also made mathematically reasonable and acceptable deductions in this stage. Three hypotheses were suggested by the groups.

<u>Hypothesis 1:</u> The doors of the lockers numbered with 1, 4, 9, 18, 35, 68, 133, 262 are open.

<u>Hypothesis 2:</u> The doors of the lockers numbered with prime numbers are always closed.

<u>Hypothesis 3:</u> The doors of the lockers numbered with perfect squares (1, 4, 9, 16,…) are open.

Validation Stage: The students started to discuss their arguments soon after they had shared the hypotheses. They were asked to provide justifications for what they thought about the truth of the statements suggested. Then the groups tried to convince the other groups about the truth of their arguments. *Institutionalization Stage*: The hypotheses which were stated and validated by the students themselves were expressed again explicitly. So the students are able to generalize the problem to 1000 lockers, or they can find out which lockers undergo two operations to decontextualize the problem and to reason further.

Conclusion and Discussion. The students endeavored to hypothesize the solution and to verify or falsify these hypotheses. Furthermore, students interacted with the milieu to reach the conclusion in an additional trial and error approach. On the other hand, group discussions gave the students an opportunity to defend their hypotheses and argue for their statements on the basis of mathematical reasoning, as well as to present their mathematical arguments. Seshaiyer, Suh and Freeman (2012) also concluded that this problem was accessible to all students and the use of models, together with acting-out strategies, seemed to engage and motivate students. In this research, however, students were made to think abstractly and create their own hypotheses. As a conclusion, it can be asserted that the students accomplished the five stages of adidactical learning situation willingly and unwittingly. The participants also expressed their opinions about their experience in the milieu, stating that they enjoyed the process more than the product and adding that this experience had broadened their horizons and made them think about their future practices in the classroom.

Keywords: Didactical situations, adidactical learning setting, problem solving, 500 lockers problem.

References

- Arslan, S., Baran, D., & Okumuş, S. (2011). Brousseau's Theory of Didactical Situations in mathematics and an application of adidactical situations. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 5*(1), 204-224.
- Arslan, S., Taşkın, D., & Kirman Bilgin, A. (2015). Effect of individual and group works on students' success in adidactical situations. *Turkish Journal of Computer and Mathematics Education, 6*(1), 47-67.
- Brophy, J. (Ed.). (2002). *Social constructivist teaching: Affordances and constraints.* Oxford, UK: Elsevier Science.
- Brousseau, G. (2002). *Theory of didactical situations in mathematics*. London: Kluwer Academic Publisher.
- Calder, N. (2010). Using stratch: An integrated problem-solving approach to mathematical thinking. *Australian Primary Mathematics Classroom*, *15*(4), 9-14.
- Cobb, P. (1988). The tension between theories of learning and instruction in mathematics education. *Educational Psychologist, 23*(2), 87-103.
- Çelik, D., Güler, M., Özüm-Bülbül, B., & Özmen, Z. M. (2015). Reflections from a learning setting designed to investigate mathematical thinking. *International Journal of Educational Studies in Mathematics, 2*(1), 11-23.
- Davies, W. M. (2009). Groupwork as a form of assessment: Common problems and recommended solutions. *Higher Education, 58*, 563-584.
- Davis, B. G. (1999). Cooperative learning: Students working in small groups. *Speaking of Teaching, 10*(2), 1-4.
- Eisenhardt, S., Fisher, M., Schack, E., Tassell, J., & Thomas, J. (2011). Noticing numeracy Now (N^3): A collaborative research project to develop pre-service teachers' abilities to professionally notice children's mathematical thinking. In S. Reeder, (Ed.). *Proceedings of the 38th Annual Meeting of the Research Council on Mathematics Learning 2011* (1-8). Cincinnati, OH.
- Empson, S. B. (2011). On the idea of learning trajectories: Promises and pitfalls. *The Mathematics Enthusiast, 8*(3), 571-596.
- Fosnot, C.T. (1996). Constructivism: A psychological theory of learning. In C.T. Fosnot (Ed.), *Constructivism: Theory, Perspectives, and Practice.* New York: Teachers College Press.
- Glasersfeld, E. (1989). Cognition, construction of knowledge, and teaching. *Synthese, 80*(1), 121-140.

- Harel, G. & Sowder, L. (2005). Advanced mathematical-thinking at any age: It's nature and its development. *Mathematical Thinking and Learning*, 7 (1), 27-50.
- Lester, F. K., & Mau, S. T. (1993). Teaching mathematics cia problem solving: A course for prospective elementary teachers. *For the Learning of Mathematics, 13*(2), 8-11.
- Ligozat, F. & Schubauer-Leoni, F. (2010). The joint action theory in didactics: Why do we need it in the case of teaching and learning mathematics? In V. Durand Guerrier, S. Maury& F. Arzarello, *CERME 6 Proceedings* (pp. 1615-1624). Lyon: INRP.
- Kaplan, J., & Moskowitz, M. (2000). *Mathematics Problem Solving.* New York: Triumph Learning.
- Kimani, P. M., Olanoff, D., & Masingila, J. O. (2016). The locker problem: An open and shut case. *Mathematics Teaching in the Middle School, 22*(3), 144-151.
- Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children* (J. Teller, Trans., & J. Kilpatrick & I. Wirszup, Eds.). Chicago: University of Chicago Press.
- Michaelsen, L., Fink, D., & Knight, A. (1997). Designing effective group activities: Lessons for classroom teaching and faculty development. In D. DeZure (Ed.), *To improve the academy.* Stillwater, OK: POD Network.
- National Council of Teachers of Mathematics (2000). *Let's Count in Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM.
- Papadopoulos, I. (2017). Opening inquiry mathematics to parents: Can they be engaged as teachers' partners in mathematical work?. *Journal of Pedagogical Research, 1*(1), 1-20.
- Patton, M. Q. (2002). *Qualitative research and evaluation methods.* Thousand Oaks, CA: Sage.
- Prawat, R. S. (1992) Teachers beliefs about teaching and learning: A constructivist perspective. *American Journal of Education, 100,* 354-395.
- Radford, L. (2008). Theories in mathematics education. A brief inquiry into their conceptual differences. Working Paper. Prepared for the ICMI Survey Team 7. The notion and role of theory in mathematics education research. Retrieved from http://www.luisradford.ca/pub/44_radfordicmist7.pdf
- Resnick, L. B. (1989). Introduction. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 1-24). Hillsdale, NJ: Erlbaum.
- Rigelman, N. R. (2007). Fostering mathematical thinking and problem solving: The teacher's role. *Teaching Children Mathematics,* 13(6), 308-314.
- Samaniego, A. H. F., & Barrera, S. V. (1999). *Brousseau in action: Didactical situation for learning how to graph functions.* Paper presented at the 4th Asian Technology Conference in Mathematics.
- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching* (pp. 334-370). New York: MacMillan Publishing.
- Seshaiyer, P., Suh, J. M., & Freeman, P. (2012). Unlocking the locker problem. Teaching children mathematics, 18(5), 322-326.

- Skemp, R. (1986). *The psychology of learning mathematics.* London: Penguin Books.
- Sriraman, B. (2004). Reflective abstraction, uniframes and the formulation of generalizations. *Journal of Mathematical Behavior, 23,* 205-222.
- Sriraman, B., & English, L. (2010). Surveying theories and philosophies of mathematics education. In *Advances in Mathematics Education: Theories of Mathematics Education: Seeking New Frontiers (pp.* 7-32). Berlin Heiderberg: Springer.
- Sriraman, B., & Törner, G. (2008). Political union/mathematical education disunion. In L. D. English (Ed.), *Handbook of International Research in Mathematics Education* (2nd ed., pp. 656-690). London: Routledge, Taylor & Francis.
- Terhart, E., 2003. Constructivism and teaching: A new paradigm in general didactics? *Journal Curriculum Studies, 35*(1), 25-44.
- Torrence, B., & Wagon, S. (2007). The locker problem. *Crux Mathematicorum, 33*(4), 232-236.
- Tynjala, P. (1999). Towards expert knowledge? A comparison between a constructivist and a traditional learning environment in university. *International Journal of Educational Research,* 31, 357-442.
- Winslow, C. (2005). Introduction: A Graduate Course on Four French Frameworks for Reseacrh on Didactics of Mathematics. In C. Winslow (Ed.) *The Didactics of Mathematics: The French Way* (pp. 7-20). Copenhagen: Center For Naturfagenes Didaktik.
- Yevdokimov, O., & Passmore, T. (2008). Problem solving activities in a constructivist framework: Exploring how students approach difficult problems. In M. Goos, R. Brown, & K. Makar (Eds.), *Proceedings of the 31st Annual Conference of the Mathematics Education research Group of Australasia* (pp.629-636). Sydney: MERGA.
- Yin, R. K. (2003). *Applications of case study research.* Newbury Park, CA: Sage.