Word of guest editors

The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry.
Bertrand Russell

Introduction

Between 26th and 30th October 2018 the Faculty of Education in Jagodina, University of Kragujevac, and the Teacher Education Faculty, University of Belgrade, organized the first international conference on the uses of the history of mathematics in mathematics education in Serbia. The conference took place in a beautiful town of Jagodina, in the southeast of the country. The training conference, supported by the European Society for the Research in Mathematics Education (ERME), involved twenty seven delegates, mainly from the countries of the former Yugoslavia's region. This special issue of Teaching Innovations, although it was inspired by some of the papers which were presented, and the work that went on during this conference, come also from our international colleagues from further apart. Let us explain this further.

The interest around the theme has existed for some time in the region, and it is important to note that some of the participants of the said conference, as well as some of the contributors to this issue (the two sets do not entirely correspond), had already collaborated over the period of the past six years, and there has already been one special issue on the history of mathematics in mathematics education published in 2014 in *Journal Teaching Innovations*[1]. We are now happy to present another issue on this theme with some old and new researchers who we have met since then.

To make the joint work more clear, we will first consider the outcomes of this first conference on the history of mathematics in mathematics education in Serbia[2], and then explain further how we put this volume together with the participants from this conference and included some of our other international researchers.

1 Lawrence, S. & Đokić, O. (eds.) (2014). History of mathematics and teaching mathematics. *Teaching Innovations*, *27*(3). Retrieved January 30, 2020. from http://www.inovacijeunastavi.rs/sr/vol-27-no-3/

2 Lawrence, S., Mihajlović, A. & Đokić, O. (eds.) (2018). *Proceedings of the Training Conference History of Mathematics in Mathematics Education*, October 26-30, 2018, Jagodina, Serbia. Jagodina: Fakultet pedagoških nauka. https://pefja.kg.ac.rs/proceedings-of-the-training-conference-history-of-mathematics-in-mathematics-education/

The background to the need for a conference

The integration of the contents of the history of mathematics is a longstanding issue in mathematics education (Schubring, 2011). Many studies revealed that when mathematics is taught with abstract formulas and algorithms without studying how these concepts developed over time, students might perceive mathematics as a set of unrelated topics (Karaduman, 2010). If mathematical theories are seen only through their final formulation, without historical interpretations, students can gain an impression that mathematics is an artificial creation, which serves only mental imagination, but has no connection to practical work or the real-life contexts (Dejić & Mihajlović, 2014). This can be overcome when students, through learning about historical facts, understand that mathematics, throughout the history, has played a significant role in the development of almost all areas of human life. Incorporating the contents of history of mathematics in mathematics classroom can help students to learn that mathematics is not something "that has arisen out of thin air" (Jankvist, 2009: 239), but is a discipline which has undergone an evolution. Integration of the history of mathematics into the teaching practice helps students understand that mathematics is not a fixed and complete system of knowledge, but that it represents a live body of knowledge that develops and changes and is closely linked to the development of both sciences and the arts (Fadlelmula, 2015; Karaduman, 2010).

We wished to examine the diverse uses of the history of mathematics in education, and in particular how this has played in the region defined by the former Yugoslavia. The mathematics in the countries in this region (post-Yugoslav) has drawn on different traditions, pre-, during, and post-Yugoslav. The question of great importance to us was to seek to understand how common threads from our joint experiences could be used to identify our current strengths, and build on our current collaborations, and consider contributions to the international networks in this area of academic research. The theme of the conference was *The History of Mathematics in Mathematics Education*, so as to enable us to discuss about the various national and international practices and give examples to enable comparisons and discussions. An overarching aim was to increase the confidence of the region's mathematics education community to engage with research and establish and build new relationships of cooperation with the communities that are nearby both geographically and in terms of traditions of mathematics education.

The participants of the conference were mathematics educators from the region of the Western Balkans who are involved into higher education institutions as teacher educators working with both primary and mathematics pre-service teachers.

Considerations on the uses of history in mathematics education during the conference

We considered the uses of the history of mathematics in teacher training and note here some of the aspects that we discussed. There are many studies that promote using history in mathematics classes, and they point at the advantages it brings. Fadlelmula (2015) points out that learning history of mathematics has many benefits for both students and teachers. For the teachers, learning history of mathematics can improve their mathematical literacy and creativity and help them better explain many of the questions they might encounter in a classroom. It also enriches their repertoire of explanations and examples, and helps them to use alternative approaches in presenting a subject or solving a problem (Tzanakis & Arcavi, 2000). Furthermore, the history of mathematics can serve a teacher as a guide through difficulties and obstacles which students face when learning about mathematical topics. Those difficulties are often similar to those which were encountered through some

historical development of certain concepts. Additionally, history of mathematics can be used as an effective tool for capturing students' attention and promoting flexibility and open mindedness in mathematics (Fadlelmula, 2015). It also helps teachers to expand their understanding of the nature of mathematical knowledge (Liu, 2003). Considering the fact that a teacher is the one who plans, prepares, and performs mathematics teaching, her/his role must not be neglected in the teaching process. To which extent and in which ways the history of mathematics will be integrated into teaching depends on beliefs, attitudes and readiness of teachers. Teacher philosophical views on mathematics as a discipline and mathematics teaching represent the most crucial factors that affect their decisions about curriculum and teaching methods (Panasuk & Horton, 2013). Apart from the knowledge of the subject and its teaching, teachers' views, beliefs, preferences about and attitudes toward mathematics influence their instructional practice (Cooney et al., 1988). Pre-service teachers often enter teacher education faculties with beliefs and attitudes that are not beneficial to the teaching of the subject (Charalambous et al., 2009). Mathematics methods courses can change pre-service teachers' knowledge, assumptions, feelings about mathematics, as well as their beliefs concerning their role as teachers in the classroom (Burns, 2010). This is the reason why teacher education programs should create opportunities for prospective teachers to develop productive beliefs and attitudes toward mathematics teaching and learning (Charalambous et al., 2009). Philippou and Christou (1998) reported that the prospective teachers' attitudes and views of mathematics showed significant change after participating in two history-based mathematics courses in teacher preparation programmes. On the other hand, some studies reported that studying history of mathematics did not have significant effect on improving students' attitudes toward mathematics (Liu, 2003) or while some dimensions of the beliefs and attitudes changed positively, others changed in the opposite direction (Charalambous et al., 2009). Some outcomes indicated that using history only for the sake of using history appeared to be superficial and impractical (Liu, 2003), and that incorporation of history of mathematics in teacher education should focus on developing useful and usable knowledge for the work of teaching mathematics. All this implicates that history of mathematics should find significant place in teacher preparation programmes.

During our conference, we undertook some research with the delegates to see how university teachers from Serbia, Bosnia & Herzegovina, Montenegro, Croatia and the Republic of North Macedonia (Western Balkans) view the role of history of mathematics in teaching mathematics. At the teacher education faculties for primary teachers in these countries, there are no history of mathematics courses or modules. Some faculties such as the Faculty of Mathematics in Belgrade have a course in history of mathematics, but there is no preparation of the pre-service teachers which would demonstrate how to integrate history of mathematics in teaching.

We therefore posed research questions with a twofold aim: we wanted to examine whether university teachers use history of mathematics in their teaching and the reasons why they use it; and we wanted to investigate their attitudes about the use of history of mathematics in the teacher education mathematics courses at their institutions.

This became formulated as two *research questions* which we posed to our colleagues (and us):

1. Do university teachers use history of mathematics in their teaching and what are the reasons for using history of mathematics?
2. What are the attitudes of university teachers about the use of history of mathematics in teaching mathematics?

Methodology of our research

Our *sample* consisted of 21 university teachers, the participants of training conference, among which 14 had a B.Sc in Mathematics and 7 in Education. The majority currently teach at teacher education faculties (18) and the rest at the faculties of mathematics (3). There were 15 females and 6 males. In terms of their working experience, we note that three participants have been working less than 10 years, 14 participants between 15 and 25 years, and four participants for more than 25 years.

The participants of the research were sent an electronic version of the *questionnaire* (see *Appendix*). The questionnaire contained two parts: the first part involved some background information (gender, years of working experience, educational background) and the second part contained two questions (one closed ended and one open-ended) and 16 Likert-scale items (6 point). The attitude Likert-scale is the most popular and we used 6 point decisions (dealt with 'neutral' mid-point). Items vary in direction to avoid response acquiescence. University teachers were asked to indicate their level of agreement with items regarding the use of history of mathematics in mathematics and mathematics education courses.

Results and discussion

As for the *first research question*, we determined that the majority of university teachers (14) use history of mathematics in their teaching. Only two participants answered that they never use history of mathematics, and five answered that they use it sometimes or rarely. We note here that the participants have opted to learn how to use history of mathematics in their work, and therefore we note their existing interest in the field and possible experience in the area. Another issue in our research was to examine the reasons of using contents of history of mathematics in their teaching work. The majority of the surveyed university teachers use the contents from the history of mathematics in their teaching to help their students to better understand the evolution of mathematical concepts, to motivate them and increase their interest for the subject, to help them to better understand and more appreciate the role of mathematics in the development of society, and to help them understand that mathematics is live course of action that develops and changes, closely related to other sciences (Table 1).

Table 1. Reasons for using history of mathematics in teaching

Items	M	SD	Cv
H1. To help students to better understand the evolution of mathematical concepts.	5,05	1,24	24,55
H2. To help students to better understand the relation between different areas of mathematics.	4,52	1,40	30,97
H3. To help students to understand historical and cultural context in which some mathematical ideas developed.	4,48	1,50	33,48
H4. To motivate students and increase their interest for the subject.	5,00	1,10	22,00
H5. To develop positive attitudes in students toward the subject.	4,62	1,28	27,71
H6. To help students better understand and more appreciate the role that mathematic had in the development of the society.	5,00	1,22	24,40
H7. To help students better understand possibilities and necessity of application of mathematics.	4,48	1,12	25,00
H8. To help students understand that mathematics is not fixed and final system of knowledge, but live course of action that develops and changes.	4,81	1,36	28,27

H9. To help students understand that errors, doubts, intuitive reasoning, discussions and alternative approaches are an integral part of mathematics in the making.	4,52	1,44	31,86
H10. To simulate some original problems and situations which were encountered by mathematicians and in which they came to their discoveries.	3,90	1,76	45,13

Similar to findings of Furinghetti (1997), we noticed that university teachers use the history of mathematics to both promote mathematics and reflect on the development of mathematical concepts. However, the teachers' answers were heterogeneous regarding the use of original problems and situations which were encountered by historical mathematicians and in which they came to their discoveries. Some authors point out that working on history or history-inspired problems, helps students to understand better the motivation behind the birth of many mathematical problems and procedures (Antónia, 2001; Tzanakis & Thomaidis, 2000). The beauty of a problem does not lie only in its mathematical problem, but even more in the reasons why the problem occurred (Fadlelmula, 2015). Moreover, using historical problems from different time periods and cultural backgrounds allows students to get to know alternative problem solving strategies (Antónia, 2001).

As for the *second research question*, results show that, in general, the attitudes of university teachers toward the use the history of mathematics in mathematics teaching at teacher education courses were positive and homogeneous (Table 2).

Table 2. Attitudes of university teachers about the use of history of mathematics in teaching

Items	M	SD	Cv
H11. All pre-service teachers (both mathematics and primary teachers) should learn history of mathematics to a certain level.	5,52	0,51	9,24
H12. History of mathematics is important and necessary in order to better understand mathematical contents.	5,14	0,85	16,54
H13. History of mathematics is important and necessary in order to better understand mathematics as a science.	5,24	0,94	17,94
H14. History of mathematics is important and necessary in order to better understand mathematics as a school subject.	5,00	0,89	17,80
H15. Knowledge of history of mathematics is useful for non-mathematicians as well.	4,62	0,92	19,91
H16. Knowledge of history of mathematics can help my students in learning and understanding mathematical concepts.	4,96	1,07	21,57

The values of coefficients of variation indicate that university teachers' attitudes are homogenous and relatively the same. All participants agree that it is important that all pre-service teachers (both mathematics and primary teachers) should get to know a certain level of history of mathematics during their studies. As for the other items, almost all participants also agree that history of mathematics has significant role in understanding mathematics as a discipline and a school subject, and that it is necessary to learn it in order for their students to better understand mathematical content.

The open-ended question asked university teachers to describe some examples of using history of mathematics in their teaching. In presenting and discussing some of these examples we used qualitative research methodology. The most common use of history of mathematics in teaching work of university teachers is to provoke interest for the subject and to motivate students for learning mathematics. They usually use stories, anecdotes or some interesting facts as introduction into some topic or to present brief history of the development and origin of some mathematical concepts.

Teacher A: I usually use stories such as those about Egyptian or Ancient Greek mathematics. When I give lectures about measurements, I talk about how ancient Egyptians measured land after annual floods in the Nile valley. When we learn about rational and irrational numbers, I talk to students about Pythagoras, his followers, their secret society and discovery of incommensurable quantities.

Sometimes it is very useful for students' motivation and for putting more effort in learning of mathematics to know that many mathematical concepts, in their earliest stages of invention, were very difficult to refine, understand and accept even by gifted mathematicians (Panasuk & Horton, 2013). This specially can be beneficial to the students who struggle learning mathematics.

Teacher B: Usually, I introduce students to the brief history of the development of concepts. I give particular attention to some 'breakthrough' points as well as difficulties encountered by generations of mathematicians. I ask of my students to find similarities in these difficulties with those that usually students face when learning mathematics.

Research reveals that when students find out that mathematicians themselves also made many mistakes, and had doubts and struggles during their inventions, they recognize the importance of hard work and determination. Some participants of our study indicated that they use stories and biographies of famous mathematicians in order to persuade their students that hard work is very important in mathematics, as well as persistence in solving mathematical problems. The previous examples represent the direct use of history of mathematics, and the reasons lie in practical methods of classroom teaching. The most prevalent use of history of mathematics in teaching is taking the history of mathematics as a ready-made collection of facts that is easy to adjust to the aims of teaching (Schubring, 2011).

Apart from the motivational functional of the history of mathematics, there are some examples of a more productive use of history of mathematics for the authentic mathematical practice. Some university teachers emphasized the relationship between the historical and methodological paths of the development of certain mathematical concepts. In this way their students, pre-service teachers, get a deeper understanding of how some concepts are formed. This genetic approach to the use of history of mathematics is particularly important for the instructional work of future teachers.

Teacher C: I make parallels between historical and didactical paths of forming some concepts, such as for example number concept. Of course, I explain to my students that pupils will not undergo centuries and centuries before they form certain concept like it was through the history of the mankind, but that didactical path will be much shorter. The historical path will help my students to understand that mental development of an individual is similar to the mental development of the human race.

Felix Klein, who promoted the genetic principle, wrote that teaching should lead young people slowly towards higher things and abstract formulations and follow the "same path on which the entire mankind struggled to climb from its naïve primitive state upwards to more developed insight" (Schubring, 2011: 82).

Teacher D: Why should the concept of set precede the concept of number? One explanation is that in historical sense, the concept of set also preceded the concept of number. To illustrate this, I talk about some native tribes who used one-to-one correspondence in order to count their sheep. They used pebbles corresponding in number to that which they wanted to count.

University teachers expressed the lowest level of agreement with the item concerning the use of some original problems and situations which were encountered by mathematicians. The reason might be the fact that majority of the participants work at teacher education faculties which educate future primary teachers and hence the level of mathematics taught is not high, meaning that presenting an original historical problem may be irrelevant. Pre-service primary teachers are exposed to mathematics contents much less than pre-service mathematics teachers. If we take into account this and that studying of original sources from history of mathematics is the most demanding and most time consuming way of using history of mathematics, the results are not surprising. However, one of the participants gave a possible example of using original problem (created by famous Serbian mathematician Mihailo Petrović Alas, 1868-1943) and described how it can be used in instruction with pre-service primary teachers. The idea of the presented example was to help student teachers to gain insight into some difficulties involved in learning geometrical concepts which are caused by a disjuncture between personal geometric knowledge derived from experience and formal geometric knowledge which derives from axioms, definitions, theorems, and proofs. Fischbein (1993) indicated that many learners have tendency to make decisions which are rather based on figural constraints than on formal geometric knowledge.

Teacher E: I will illustrate an example created by Mihailo Petrović. An angle AFB is given (see Figure 1). Points A and B belong to the sides of the angle. Let's draw the perpendicular lines to the both sides of the angle AFB through the points A and B. The point of intersection of those perpendicular lines we will mark with C. Let's draw circle that passes through the points A, B and C. The points of intersection of the circle and sides of the angle FA and FB we will mark with D and E respectively. Since the angle DAC is right angle, it follows that segment DC is diameter of the circle, and the midpoint O of this segment is the center of the circle. Similarly, since the angle EBC is the right angle, it follows that EC is another segment of the circle, and the midpoint of that segment O' is the center of the circle. It seems that the circle has more than one center. The conclusions that were based upon the picture seemed to be right. How is this possible? Petrović explains this in two ways: (1) We used an incorrectly drawn picture and by reasoning we came to incorrect results; (2) When drawing pictures, we need to take care of accuracy.

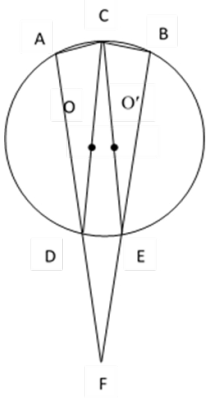

Figure 1. An incorrectly drawn image according to Mihailo Petrović Alas (Dejić, 2001: 618)

Many mistakes which students make in their geometrical reasoning can be explained by this kind of split between the conceptual and the figural aspect of the geometric figures (Fischbein, 1993). Students tend to consider geometrical objects as material objects and specific pictures, rather than theoretical, ideal objects with particular properties (Đokić & Zeljić, 2017). This fact results in the phenomenon of students trying to solve geometrical problems by relying on their visual perception instead of mathematical deduction based on the properties of those geometrical objects (Đokić, 2017).

Conclusions from our conference discussions and the research results

At the close of the conference, we compiled all the results and did our analysis as shown above. We were then able to make some conclusions about our practices and experience up to this point. In order to prepare prospective primary and mathematics teachers to use history of mathematics in mathematics teaching, we found that it was necessary that they should be exposed to some history of mathematics contents during their university education. The results of our research indicate that university teachers participating at our conference expressed positive attitudes toward the use of history of mathematics in mathematics teaching. The majority of participants reported that they use history of mathematics in their instructional work, and the most common reasons they provided were that this helped their students to understand better the evolution and development of mathematical concepts, an increasing interest in the subject, and motivation. On the other hand, the use of original problems and situations which were encountered by mathematicians in the history of the discipline were less prominent – something that may be in the focus of our further research.

Although we are aware that the findings of this analysis and study, which came from our conference, cannot be generalized due to the small sample size, we find them relevant and important for further investigations and improvement of mathematics teaching and learning at the institutions which educate future primary and mathematics teachers.

Our training conference represented a great opportunity for mathematics educators from the region of the former Yugoslavia's teacher education faculties to share their experience and discuss various national and international practices. The conference enabled participants to present and compare various approaches and ways of incorporating the history of mathematics in mathematics and didactics of mathematics courses exploring historical traditions in mathematics education.

The contributions in this edition

This special issue of the *Teaching Innovations* journal includes not only theoretical and empirical research papers on history of mathematics and its relationship with mathematics education, but also papers dealing with some methodological and developmental issues. The role of history or epistemology of mathematics is reflected at the primary, secondary, and tertiary levels, and in the pre-service primary and mathematics teacher education, from cognitive, pedagogical, and philosophical points of view.

The papers are organized in three groups. In the first group we have presented the papers dealing with topics related to the history of mathematics and how their research and presentation to the student teacher population may benefit the learning of mathematics. This group of papers begins with a paper by Đokić, Jelić, &

Ilić on the topic referring to the history of geometry and is followed by the paper by Massa Esteve on the topic of the use of original sources in learning algebra.

Based upon the Fischbein theory of figural concepts, **Đokić et al.** examine the pre-service primary school teachers' geometric reasoning regarding the correlation between figural (pictoral) and conceptual properties of geometric objects (angle and cube). In this paper, the authors find different examples from the history of geometry, theorems of Eudemus of Rhodes and Thales of Miletus and elaboration of these theorems in work of Serbian mathematicians Mihailo Petrović Alas, and put them in mathematics education context and didactical situations in which an image or non-prototype example creates a conflict that draws upon a real conceptual understanding of an object, i.e. the image of an object dominates the formal definition.

The second paper in this group, by **Massa Esteve**, offers an investigation of the ways of integrating original sources in Mathematics History course for the bachelor's degree in Mathematics, and their educational effects. Based on the classroom experience, Massa Esteve points out that the implementation of historical activity helps pre-service teachers to gain a better understanding of the role played by the relationship between algebra and geometry in the development of mathematics, and this improves their mathematical training.

Our second group of papers is focused on the development of mathematical concepts through the history of mathematics. In this group we have different approaches: two papers on the development of mathematical artifacts in many diverse contexts by Lemonidis & Gkolfos and Maričić & Lazić, and two on the use of mathematical methods by Mihajlović, Vulović, & Milikić and Pjanić.

Lemonidis et al. chose to look at the history of number line and its application in the classroom. They make parallels between the historical development of the concept and the idea of number line and the cognitive development. This paper therefore deals with epistemological and educational issues, as well as pupils' cognitive processes and difficulties.

Maričić et al., on the other hand, chose to study the history of abacus in different cultures and contexts and show its interpretations and uses. In their paper, the authors draw attention to the abacus, an ancient computing tool, by exploring its role, development and its uses through history of mathematics. The authors point out the possibilities, benefits, and opportunities that abacus offers to mathematics teaching and learning.

The paper written by **Mihajlović et al.** on the use of Fengcheng method in Serbian mathematics classrooms is also in this group. The authors demonstrate how they designed teaching and learning materials based on the sources from the history of mathematics, and investigate the effects of its use in educational settings at primary level. The authors teach primary school students some pre-algebra concepts by using the ancient Fangcheng method for solving systems of linear equations. An interesting paper for many reasons, not only because it can be argued that this type of research, apart from exposing children to different cultures and contexts, should be a part of any national mathematics curriculum.

Pjanić, on the other hand, traces Abu'l-Wafa's problem and its application to (historical) practical crafts. This paper also deals with the use of episodes of history of mathematics in classroom, as did Massa Esteve from the first group. In her case study however, Pjanić exposes a group of pre-service mathematics teachers to the history of mathematics as the source of problems upon which we can create a teaching process. The author shows that using some episodes from the history of mathematics, such as Abu'l-Wafa's problem, improves pedagogical content knowledge of future teachers of mathematics.

Finally, our third section of papers focuses on mathematicians from a historical perspective. Here we have ordered papers from particular to more general ones. The first paper, written by **Dejić**, analyzes one

mathematician's contribution not only to mathematics, but to mathematics education. Dejić talks about Mihailo Petrović Alas, the most famous Serbian mathematician, and gives us some examples from Petrović's educational work. Dejić documents some of Alas' efforts to show how secondary students might be motivated for learning mathematics through some geometry tasks related to the use of the Pythagoras' theorem.

Fan & Li contributed a highly engaging study on the presentation of mathematicians in Chinese mathematics textbooks, the three series from Grade 2 to Grade 9 (primary and junior secondary schools in China). Their research has provided some very interesting data through which the authors show how the examples of historical mathematicians make their 'appearance' in the present-day Chinese mathematics classrooms via mathematics textbooks.

Finally, **Lawrence** discusses in her paper an example of how teachers can create real and imaginary spaces for their learners, so that they can experience mathematics from the past. In this paper the author explores the relationship between history, philosophy of mathematics, and the teaching of mathematics. Lawrence looks at why one should use the history of mathematics, how both teachers and students might profit from studying it, and how teachers can incorporate it in their teaching work. By reflecting on her own experience, both as a teacher and mathematics historian, the author introduces a simple, but effective, ancient method which consists of creating an imaginary theatre from mathematical objects and images.

In making our selection of papers for this special issue of the *Teaching Innovations* journal we tried to bring out the variety of ways researchers approach problems and the possibilities of incorporating the history of mathematics in mathematics education. We hope that you will find all the papers in this collection engaging and important for your practice in mathematics education. Furthermore, we strongly believe that the work, such as the one presented to you here, has helped to increase the confidence of our local mathematics education community to engage with research and establish (and re-establish) relationships with the international communities both those nearby, and with our colleagues from around the world. We hope that this issue, which is the second special issue dedicated to the history of mathematics in mathematics education, will become a common practice in the *Teaching Innovations* journal.

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Appendix

Questionnaire

This survey is designed to gather information about the use of history of mathematics in teaching at higher education institutions which educate future primary and mathematics teachers. In the first part of the questionnaire you are asked to supply some information about your academic and professional background, and courses that you teach. The second part of the questionnaire contains items related to the perceptions and beliefs about the use of history of mathematics in teaching. The survey takes 15-20 minutes to complete. Your answers will remain confidental and will be used for the purpose of the scientific research. Please take a moment to complete the survey below.

Thank you for your cooperation!

I

1) Gender (**bold**) M F

2) Years of teaching experience _____

3) The faculty you graduated from (**bold**):

a) Natural Sciences and Mathematics b) Teacher education

c) Some other (write which one) _____

4) The faculty you are currently working at (**bold**)

a) Natural Sciences and Mathematics b) Teacher education

c) Some other (write which one) _____

5) Please list all the courses that you teach:

II

1) During the studies I had the opportunity to become acquainted with the content of the history of mathematics (**bold** one of the listed answers):

a) Yes,

b) No,

c) Something else (fill) _____

2) In my work with students, I use the content of the history of mathematics (**bold** one of the listed answers):

a) Yes,

b) No,

c) Something else (fill) _____

3) If there is a mathematician who has made a special impression on you with her/his life and work, please state who she/he is and briefly explain why.

4) The following items apply to the courses that you teach. Please circle the number on a scale from 1 to 6 (1=Totally disagree, 6=Totally agree) to indicate the extent to which you agree with the statement.

	Totally disagree					Totally agree
1. To help students to better understand the evolution of mathematical concepts.	1	2	3	4	5	6
2. To help students to better understand the relation between different areas of mathematics.	1	2	3	4	5	6
3. To help students to understand historical and cultural context in which some mathematical ideas developed.	1	2	3	4	5	6
4. To motivate students and increase their interest for the subject.	1	2	3	4	5	6
5. To develop positive attitudes in students toward the subject.	1	2	3	4	5	6
6. To help students better understand and more appreciate the role that mathematic had in the development of the society.	1	2	3	4	5	6
7. To help students better understand possibilities and necessity of application of mathematics.	1	2	3	4	5	6
8. To help students understand that mathematics is not fixed and final system of knowledge, but live course of action that develops and changes.	1	2	3	4	5	6
9. To help students understand that errors, doubts, intuitive reasoning, discussions and alternative approaches are an integral part of mathematics in the making.	1	2	3	4	5	6
10. To simulate some original problems and situations which were encountered by mathematicians and in which they came to their discoveries.	1	2	3	4	5	6
11. All pre-service teachers (both mathematics and primary teachers) should learn HISTORY OF MATHEMATICS in a certain level.	1	2	3	4	5	6
12. HISTORY OF MATHEMATICS is important and necessary in order to better understand mathematical contents.	1	2	3	4	5	6
13. HISTORY OF MATHEMATICS is important and necessary in order to better understand mathematics as a science.	1	2	3	4	5	6
14. HISTORY OF MATHEMATICS is important and necessary in order to better understand mathematics as a school subject.	1	2	3	4	5	6
15. Knowledge of HISTORY OF MATHEMATICS is useful for non-mathematicians as well.	1	2	3	4	5	6
16. Knowledge of HISTORY OF MATHEMATICS can help my students in learning and understanding mathematical concepts.	1	2	3	4	5	6

5) Please list all the courses you teach in which you use history of mathematics contents:

6) If you use history of mathematics contents in your work with students, could you please describe one specific situation/example?

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