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Scientific review paper

Abacus computing tool – from history to application in mathematical education[2]

Summary: *Bearing in mind that the history of mathematics and its discoveries can play a significant role in mathematics education today, the authors draw attention to an ancient computing tool – abacus, and point out its role, importance, values, and opportunities it offers in working with children. In order to achieve this goal and using the method of theoretical analysis, the authors researched the values of application of the Japanese abacus (soroban) in mathematics education through the following aspects: learning mental arithmetic, understanding the place value of a digit, contribution to working with children with developmental disabilities, primarily with visual and hearing impairments, as well as contributions to children's mental development and problem-solving abilities. Based on numerous research papers and views, the authors conclude that Japanese abacus can have many benefits in working with children in mathematics education, among which the most important are conceptual understanding of mental arithmetic and developing mental computation and problem-solving skills, gaining a clear mental picture of the structure of numbers, their magnitude and relationship, understanding the place value of a digit, but also developing motivation and positive attitudes toward mathematics.*

Keywords*: abacus, history of mathematics, soroban, mental computation, mathematical education.*

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Introduction

Numbers, counting, and arithmetic operations were propaedetically formed in humans in the earliest epochs of civilization, and they have developing in parallel with the general progress of human consciousness. The beginnings of mathematics and mathematics education can be traced back to the earliest stage of human development (Špijunović & Maričić, 2016). Mathematical knowledge was developed spontaneously and instinctively, in the process of work and play, and was passed on from older to younger generations. Mathematics, during this period, arose primarily from the practical needs of calculating, counting, and measuring. The beginning of writing, in approximately 3rd millennium BC, created an assumption that basic mathematical knowledge was also written down. The first written "monuments" of mathematics made in the areas settled by great civilizations, testify to the mathematics of that time. The early period of mathematics development is characterized primarily by *practical arithmetic*, that is, the gradual development of the process of calculating and the possibilities of its application (Lazić, 2014). For the purpose of computing, various calculus "tools" were created, among which abacus certainly stands out as the most famous one.

An ancient computation device, called *abacus* (from the Greek word *abak* - writing board, calculus) is the oldest known manual calculating tool in history, used as early as in Mesopotamia, Egypt, ancient Greece, and Rome. This computing device appeared around the world at different times, and the advanced abacus is still used today for everyday computation by people in Russia, China, and Japan.

Starting from the fact that "it is very useful to find out the true origin of extraordinary inventions that appeared not by chance but by the power of human thought" (Gerhardt, 1858: 392), and that it is "necessary for a man to know what provoked the development of mathematical ideas, which methods of study were used in the past and how the problems that were posed were solved" (Dejić & Mihajlović, 2014: 16), this paper draws attention to the abacus, pointing out its importance, values and opportunities that can be realized in working with students within mathematics education.

Abacus – origin, development and application possibilities

The first forms of abacus are about 2500 years old. Over time, depending on the people and the numeral system used (China, Japan, Russia, and India), the abacus was modified to gain sophistication. The original appearance of the abacus was based on stone, clay or wood with sand, on which people used to write (Moon, 1971). The oldest preserved calculating board was made of white marble, with pebbles being placed in the sand grooves. Found in 1864 on the island of Salamis by which it was named, it is housed in the National Museum of Epigraphy in Athens (Heffelfinger & Flom, 2004; Samoly, 2012). The development of society entailed the need for a portable computing aid. It is believed that the Romans were the first to design a portable abacus – a board made of bronze, in whose grooves they put pebbles and thus wrote numbers (Figure 1). Their position on the board corresponded to a specific place value, which represented the end result (lat. calculus – pebble, lat. calculare – calculate) (Heffelfinger & Flom, 2004; Škorvaga, 2011).

Figure 1. Roman abacus (Škorvaga, 2011: 2).

The Chinese abacus, named *suanpan*, appeared in the 14th century. It was modeled after the Roman abacus, and adapted to a decimal and hexadecimal numeral system. The *suanpan* consists of a rectangular wooden or metal frame with 13 vertically mounted rods of a certain place value, with the beads attached along the rods (Figure 2). Inside the frame there is a horizontal bar dividing the abacus into two unequal parts, so that the rods in the upper part have 2 beads whose numeral value is 5 each, while the bottom parts of the same rods have 5 beads with numeral value 1 (Kojima, 1954; Bernazzani, 2005; Banzić, 2017).

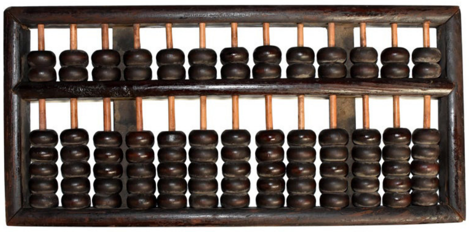

Figure 2. Chinese abacus suanpan (Škorvaga, 2011: 2).

In the 16th century, a *soroban* abacus was created in Japan, simpler in structure and easier to use than the Chinese, intended solely for the decimal numeral system. Unlike the Chinese abacus, *soroban* has one bead on each rod above the bar in the upper part, and 4 beads on each of the rods bellow the bar. Each bead in this abacus below the bar has the value of 1, while a bead above the bar has the value of 5. (Figure 3). Only when the beads touch the bar does their value count (they move from zero position to a number). Looking at the abacus from left to right, the last rod represents the row of units, the rod next to it - the row of tens, the next one - the row of hundreds, then the row of thousands, etc. (Banzić, 2017).

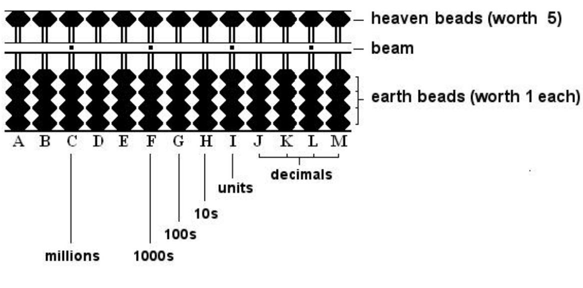

Figure 3. Japanese abacus soroban (Samoly, 2012: 61).

In the 17th century, the Russians designed their own abacus, called *schoty*, consisting of a rectangular frame, inside which there were horizontally placed rods with beads, without a bar (Figure 4). There are 10 beads in each row, and for the ease of calculation the fifth and the sixth are colored in a different color (Heffelfinger & Flom, 2004). This counting tray is the forerunner of a didactic tool that also found application in the education system. In schools in the Republic of Serbia, we can still find plastic counting trays with horizontally positioned metal wires with multicolored plastic beads (10 beads on each wire).

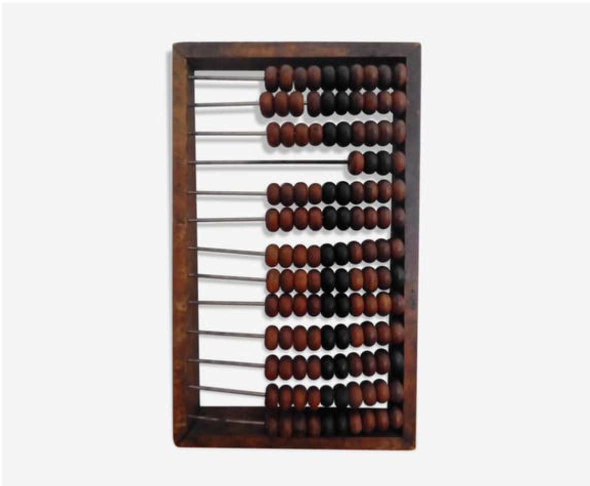

Figure 4. Russian abacus schoty (Škorvaga, 2011: 2).

Modeled on the Japanese abacus, Indian people introduced the latest version of the mechanical abacus, which they later designed in digital form (Figure 5). The benefits of this abacus are reflected in the neutral background color with contrasting colored beads, thus facilitating students' mental computation (Ahamed, 2012).

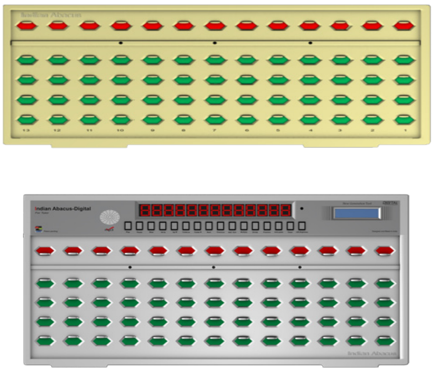

Figure 5. Indian abacus (mechanical and digital) (Ahamed, 2012: 1).

For years abacus has been an important tool for performing computational operations: addition, subtraction, multiplication, division, square root, cubic root, etc. For a long time it was an integral part of the mathematics education in Asian schools. Even the success that students from these countries achieve in international competitions is related to the use of abacus and the effects it has on their cognitive development, mental computation, and conceptual understanding of mathematics (Starkey & Klein, 2008).

In addition, due to its universality and simplicity, the abacus has served as an inspiration in art (in the area of design) and science (an atomic abacus whose use is based on a microscope and a diamond needle that moves beads/molecules) (Škorvaga, 2011).

The technological revolution and the development of modern society have inevitably brought about the neglect of this, until recently, indispensable tool in mathematics education. Today, abacus is rarely used as a tool in mathematics education. Perhaps this is due to the great advancement of modern didactic tools, digital tools and technologies which suppressed ancient didactic aids. However, we have recently witnessed the organization of numerous programs for children, which base their activities on developing the ability to perform calculus operations on the Japanese abacus, in order to encourage the development of mental arithmetic, and thus cognitive development, concentration, persistence in work and love for mathematics. In this way, we return to the history of mathematics, take valuable things from it, revive it and put it into practice, whereby "old ideas and mathematical creations suddenly resurrect and take new forms" (Dejić & Mihajlović, 2015: 68).

We have witnessed that throughout the history of education, and especially mathematics education of children, numerous authors (F. Frebel, M. Montesori etc.) have created didactic tools aimed at developing mathematical concepts and understanding mathematics. It was on this basis that the abacus was created, as the first computer, in order to enable learning of arithmetic and its conceptual understanding. If we recognize that "the history of mathematics is the basis of modern scientific methodology and is one of the most important sources of thought processes" (Dejić & Mihajlović, 2015: 67), then it is worth considering the application, role, importance and value of abacus and its use in working with children within mathematics education, as well as outside of it. Using theoretical analysis, this paper focuses on exploring the importance of applying the Japanese version of abacus (*sorban*), which is considered to be the most effective, given that it is adapted solely to the decimal numeral system. The study covered the following aspects:

- the application of abacus in learning mental arithmetic and understanding the place value of a digit;
- the application of abacus in working with children with developmental disabilities, especially visual and hearing impairments;
- the value of abacus in the mental development of children,
- the value of abacus in encouraging problem solving.

Abacus – a tool for learning mental arithmetic

The backbone of mathematics teaching programs today is the content of arithmetic. It is gradually adopted and developed in the learning process, starting with pre-school mathematics education and slowly expanding through blocks of numbers to 5, 10, 20, 100, 1000, set N, etc. The stability of teaching arithmetic and its outcomes are crucial for all other contents of algebra and geometry, which naturally build on arithmetic and form the basis for their introduction. Mastering computational techniques through the mathematical education program takes place on two levels: mental (oral) computation and written computation. In recent years, special emphasis has been placed on the importance of mental arithmetic, as a significant part of the elementary school mathematics program, and there has been an increasing recommendation for mental arithmetic to become an integral part of that program (Maclellan, 2001; Reys & Barger, 1994; Verschaffel et al., 2007; Zeljić et al., 2017). A dominant feature of mental arithmetic is flexible adaptation of strategy to the structure of problems based on an essential understanding of computation procedures and a well-developed sense for numbers (Verschaffel et al., 2007). Mastery of mental computation skills is an important element of mathematical literacy, a tool that is suitable for application in everyday life as well as in problem solving procedures, as it contributes to focusing attention on the problem situation and problem solving (Zeljić et al., 2017; Rubenstein, 2001). On the other hand, one should be aware of the dangers of the early introduction of formal computation procedures which can lead to misunderstanding of the number as a basis for mental computation strategies (Anghileri, 2001).

Numerous researchers have highlighted the importance of mental computation. Thus, it is emphasized that this form of computation enables students to understand number structure, learn how numbers work and develop cognitive and metacognitive thinking (Heirdsfield & Lamb, 2005) and provides opportunities to "develop the ability to make estimations in computation and contribute to a deeper understanding of the concept of number" (Zeljić et al., 2017: 49) and the decimal system (Carpenter et al., 1998; Fuson et al., 1997; Threlfall, 2009; Zeljić et al., 2017). Reys and Barger see it as a "*vehicle* for promoting thinking, conjecturing and generalizing based on conceptual understanding" (Reys & Barger, 1994: 31). Of particular importance to mental computation is cognitive thinking, since "first, it influences the ability of children to solve a problem; second, it develops creative thinking and third, it attracts interest of children" (Ahmad et al., 2010: 73).

Mental arithmetic is defined as the act of adding numbers together, multiplying them and the like, in your mind, without writing them down (Longman, 2010, according: Ahmad et al., 2010: 72). The starting point for mental computation is to understand the decimal numeral system, that is, the conceptual understanding of the number and value of a digit in that number. The place value is "one of the most important arithmetic concepts to be learned by children in early elementary grades" (Sharma, 1993: 3). Research shows that there is a connection between children's understanding of the notion of number and their capacity for mental computation, while this link with the ability to written computation is negligible (Carpenter et al., 1998; Van den Heuvel-Panhuizen, 2001; Verschaffel et al., 2007; Linsen et al., 2015).

The methodological approach that should lead to a conceptual understanding of the notion of number has to be based on concrete, realistic, obvious means that lead to a clear idea of the number and value of each digit in that number. It is exactly the conceptual understanding of the digit place value that is provided by abacus. It is a useful educational material that allows students to see easily not only the number of ones and tens but also their relative position and contributes to understanding the place value of the digit (Amoah et al., 2019). An early understanding of the notion of number and the place value of a digit is essential for developing the basis for mental computation (Moeller et al., 2011). The abacus is a tool that can help in the materialistic representation of double digit or multiple digit numbers (Lemonidis, 2003, according to Baralis et al., 2012: 3; Lemonidis, 2015). How is that performed? Looking at the abacus from right to left, the first rod represents a row of units, the row next to it is a row of tens, then a row of hundreds, followed by thousands etc. (Lemonidis, 2003, according to Baralis et al., 2012: 3). Students are introduced to the place value of each digit of a number and thus gain a deeper understanding of the structure of the number with the help of beads. By physical, non-symbolic representation of the number on the abacus, they visualize the position of the place value of the digit in a certain number, which contributes to better understanding of the number structure. For example, the number 471 would be displayed on abacus as follows: looking at the abacus from right to left, in the third row, the row of hundreds, four beads are raised in the field below, each having the number value of 1. Then, in the second row, the row of tens, 3 beads are raised in the following way: in the field above the bar one bead, whose number value is 5, is lowered, while on the same rod, in the field below the bar, 2 beads are raised, where each has a value of number 1, which makes 7 tens in total. In the first row, the row of units, one bead is raised in the field below the bar. In this way, with the synergy of the senses of touch and sight, the notation of number 471 is presented non-symbolically, giving a clear mental picture of the number structure, which leads to a deeper, conceptual understanding of the number.

After the first step, learning about the value of numbers on the abacus, we move on to performing computation operations with numbers. Computation on an abacus involves the activity of four fingers (thumb and forefinger of right and left hands). It was precisely J. Piaget who emphasized that during the process of development of form perception, the hand plays an important role as an organ of touching and acting on the object. In observing shapes, sizes and computation procedures, tactile perception is considered to be significant and to lead over visual in the period of initial mathematics education. During this period, the hand "teaches" the eye. The touching hand helps organize coordinated monitoring of the contours of the subject with gaze. Exercising through specific manipulative activities with fingers helps to remember the movements that came to the solution, which speeds up the solution of each following example. The research conducted by *P. I. Zinchenko* and A. G. Ruske found that when solving problems, using hand and eye alternatively, students adopt a system of sensory standards, which significantly stimulates development of observation of computation procedures in initial learning (Prentović & Sotirović, 1998). Gracia-Bafalluy and Noel (2008) drew attention to the role of hand in understanding numbers stating that "an interesting phenomenon that finger training, which requires good finger differentiation, could increase young children's numeral performance, suggesting a functional link between finger gnosis and number skills" (Gracia-Bafalluy & Noel, 2008, according to Hu et al., 2011: 11). Enabling children to use the abacus begins by performing physical activities that involve moving beads with both hands at the same time. Then, a simulation of hand movements in the form of actual hand movements is practiced, imagining the movement of beads on the abacus, where, finally, the computation is performed on a mental level, without actually moving the fingers.

Abacus works by "initially learning to perform a physical abacus with both hands simultaneously, and then practice to simulate abacus operation in mind with actual finger movements as if they could manipulate the imagined abacus beads. Eventually, they can manipulate numbers via an imagined abacus in their mind without actual finger movements" (Hu et al., 2011: 11). Children can "start counting with imagination in their brain, then movements to the real fingers" (Cheah, 2006, according to Ahmad et al., 2010: 72). This method shows that children are able to calculate quickly and accurately. The visual representation of an abacus "such as the arrangement and colour help children understand the structure of numbers in an easy and effective way and avoid a reliance on counting" (Tan et al., 2009: 1009).

Mental abacus is "a specific type of mental calculation when the individual imagines or visualizes rows of abacus beads while solving problems". Vasuki also sees the value of abacus, above all, in a greater ability to compute compared to non-mental computation (Vasuki, 2003).

Research studies show that numeral ability in children who used abacus has been significantly improved (Hatano & Osawa, 1983). Mental arithmetic is "a special form of computation, which involves mental visualization of the abacus and simulation of movement whereby the correct solution is quickly reached without using any aids" (Banzić, 2017: 17). Today, abacus is used to develop mental strategies that allow abandoning specific activities and transfer to the mental plan of computation with numbers. Based on the mental image of the abacus that a student imagines during the learning process, he/she *"sees"* the structure of the number used in computation, and the stimulation thus induced turns into a reaction in the cognitive area, in the form of an automated, fast and efficient mental arithmetic, whereby the student constantly performs mechanical operations thoughtfully, imagining, visualizing the movement of the beads on the abacus in the head. Such a higher level of computation involves computing without directly handling the abacus, but by using mental representation of the calculus tool and the thoughtful execution of operations on it. The abacus counting takes place through non-symbolic numbers. First, children transform symbolic numbers (Arabic numerals) into non-symbolic ones, whether in a physical or mental abacus, and then re-count them from non-symbolic to symbolic numbers.

The stated values of the abacus and its applications are confirmed by research studies showing that students who used the abacus in training to count have more developed representations of mental computation (Stigler et al., 1986), they count more quickly (Miller & Stigler, 1991) and have a more developed arithmetic ability compared to those who did not receive mental abacus training (Chen et al., 2006; Huang et al., 2015; Wang et al., 2015). It is particularly important that as a teaching aid, the abacus does not result in high levels of dependence (Tan et al., 2009).

Abacus – a tool that stimulates mental development

Learning mathematics and the development of abilities, above all abstraction, generalization, reasoning and other thought operations, take place simultaneously and it is impossible to separate them. Mathematics plays a big role in mental development because all activities within mathematical education are reduced to mental activities. The success of performing mental activities, the ability to abstract, largely depends on the basis on which such learning takes place. All activities have to move from practical-perceptual thinking to the level of conceptual-logical thinking. It is the abacus that forms the basis which, in the opinion of numerous researchers, contributes to mental development of the child, because in the beginning of the learning process, it creates a concrete visual and manipulative background. Studies have shown that abacus develops not only mathematics calculation, but also develops memory consistently (Bhaskaran et al., 2006). Using abacus "sharpens our memory and increases our ability to perform mental calculations" (Sarvari et al., 2015b: 125). The same views are expressed by other researchers (Chen et al., 2006; Tanaka et al., 2002). It is Chen and associates (2006) who in their research show that the changes in the brain occur with intensive training and practice on the abacus.

Abacus also helps in the visualization process, which directly reflects on mental development. The fast communication that takes place between hands and brain stimulation contributes to the rapid and balanced overall brain development (Bhaskaran et al., 2006). Abacus learning is an improvement of numeral memory, as well as an improvement of memory in spatial arrangement (Amaima, 2004).

It should be noted that abacus produces effects in mental development, primarily because of mental computation, which encourages mental activity. Mental computing takes place in such a way that concrete actions are visualized on the mental plane, and this plays a key role in computation. This type of learning arithmetic contributes to improved visualization, concentration, memory and analytical skills and patience being developed through exercise and working on abacus (Vasuki, 2013).

Abacus – a tool for working with children with disabilities

In addition to the above mentioned application and pointing out the importance of abacus in working with children with normal development, we want to draw attention to the role of abacus in working with students with developmental disabilities, especially those with visual and hearing impairments. These students require special attention and special forms of support in learning mathematics. Both categories have a disadvantage related to the basic form of communication - language, whether spoken or written. Children with visual impairment have a problem with written expression, and children with hearing impairment in speech, and for this reason, in working with them, it is necessary to create special conditions for learning in order to allow them to conceptually understand the number and mental arithmetic and enable them to use mathematics in solving the real-life problems.

Abacus is regarded by many authors as a powerful tool in teaching children with visual impairment and the results show that these students achieve good learning outcomes if they use it (Amato et al., 2013; Brawand & Johnson, 2016; Jadhav & Gathoo, 2018; Matías-Guiu et al., 2016; Vita & Kataoka, 2014). There are also recommendations that learning mathematics in this category of students, in addition to Braille, should be based on the use of abacus and that this type of learning should be started as soon as possible (Amato et al., 2013). For a child with a visual impairment, abacus means the same as paper and pen for a child without this problem. Visually impaired children experience all relationships on mental level, as hands can only explore one object and transfer it to the mental plane. These children do not have the ability to see the illustration presented in a textbook or other source, but through tactile perception, they transfer concrete actions to the mental plane and learn to perform computation operations with numbers. By manipulating the abacus, these children visualize concrete activities with numbers and transfer them to the mental plane, thus developing mental computing skills.

The situation with children with hearing impairment is similar. These children have a major deficiency, which is, first and foremost, related to their primary hearing impairment, which is reflected in a lack of language and communication with the language. This is especially noticeable in learning mathematics. In addition to mathematics being characterized by a specific language, it is also a key element in defining the meanings of the terms, but also to success in mathematics. Research shows that the use of abacus, which in this situation through kinesthetic action, aids visualization of mental computation and achieves significant results in working with this category of students (Jadhav & Gathoo, 2018: 63).

Abacus – a tool that encourages problem solving

Problem solving skills are an important outcome of mathematics education. In order to solve the problem, we first have to master arithmetic at the level of deep conceptual understanding. Mental arithmetic has "an important place in teaching and learning mathematics, and above all in the development of problem-solving skills" (Zeljić et al., 2017:

49). The mastery of mental computation is the basis for problem solving. An individual should be able to count mentally, as this is an important ability for dealing with everyday, real-life situations, especially given that written computation is tied to paper and pencil. Mental computation and problem solving are complementary processes. While performing mental computation, a student solves the problem. Given that we have highlighted the importance of abacus for mental computing, its impact can be transferred to both problem solving and capacity building in this sphere. The transfer is reflected, on one hand, through the visualization of the problem, which is the basis for mental computation, and on the other, through flexibility in the choice of computing strategies. It is the visualization of the problem, its transfer to the mental plane and the choice of strategy in mental computing that is a condition for the successful problem solving.

Numerous research studies show that training to use abacus and abacus computation itself significantly contribute to improving problem-solving abilities (Lean & Lan, 2005; Amaima, 2001). Abacus is “practical, useful in solving mathematical problems” (Tiwari et al., 2017: 61) and “abacus skill causes to coordinate visual, auditory and sensory inputs to solve the problems by analyzing of visualization of beads by the brain” (Shanthala , 2011: 25). Rubenstein (2001) also points to the importance of abacus in problem solving, which he explains by the flexibility in choosing a strategy to solve. In their study, Lean and Lan conclude that “the mathematical problem solving abilities among pupils who learn abacus-mental arithmetic is higher compared to pupils who do not learn abacus-mental arithmetic” (Lean & Lan, 2005: 5). Research papers (Lee et al., 2007; Shwalb et al., 2004; Freeman, 2014) indicate the importance of abacus and work on it to develop motivation and positive attitudes toward mathematics.

Conclusion

History of mathematics contributes to encouraging motivation of students to learn mathematics content and increasing student involvement in the process of learning (Lazić & Maričić, 2019). This paper may be put in the context of the above quotation, for, through theoretical analysis, the authors wanted to draw attention to the ancient calculating device, created many years ago, which still lives today and is used; more importantly, it is the subject of numerous studies. The fact that the abacus is alive, though not used in formal education, can be explained by the words of M. Dejić and A. Mihajlović, who emphasize: “Unlike other human activities (art, social sciences), in mathematics, old and new are closely related, they are the links of the same chain” (Dejić & Mihajlović, 2015: 72).

Although abacus was created a long time ago with the original purpose of being a tool to assist in computation, its benefits in mental and mathematical development have surpassed its values and have become an inexhaustible universe that lives on in present times. Modern age and technological revolution broke up with history and abacus has been replaced by electronic calculators; yet, abacus connoisseurs are always resorting to this device and keep reviving it. Although not represented in the formal mathematics education, this tool is “alive” and used in numerous programs offered to children.

The contribution of the Japanese abacus to learning by understanding and as an aid to computation is multiple: gaining a clear, vivid idea of the structure of numbers, their magnitude and relationship, a better understanding of the place values of digits, visualization of arithmetic operations and basic computation procedures, effective development of mental computing skills, developing mental abilities, and problem solving and motivation. In addition, by operating abacus, the necessary knowledge is acquired quickly and easily, through the synergy of all senses, in an interesting and fun way. The study and use of abacus with all senses involved make this tool an ideal aid for the effective development of mental arithmetic.

The paper highlights some aspects of the Japanese abacus application in working with students. It draws attention to this ancient calculating device and the benefits gained in working on mental computation, conceptual understanding of arithmetic, problem solving, and mental development. Being aware of the above, we justifiably assume that the stated views and conclusions of numerous studies can serve as a basis for further broader research of the benefits of abacus application in mathematics education, without disputing the importance of the program contents of elementary mathematics teaching, which cover standard algorithms and computation procedures.

Finally, one should be aware of a number of limitations when it comes to training to work with abacus. It takes a long time to use and apply the abacus, especially in the beginning, and it requires a lot of persistence and hard work by its users. Despite the fact that the practice of mathematics education in schools today does not recognize the use of Japanese abacus in working with children, this does not diminish its importance and values. There are numerous opportunities for the organization of special extra-curricular activities in schools aimed at working on abacus. Although this study points to the great importance and role of abacus use for schoolchildren in learning mathematics, studies addressing this problem in Serbia are quite rare.

References

- Ahamed, B. (2012). *Abacus: The next generation. Brain Education & Training Skils*. Retrieved September 22, 2019. from www: https://www.indianabacus.com.
- Ahmad, S. Z., Rosmani, A. F., Ismail, M. H. & Shakeri, M. (2010). An Introductory of Mental Arithmetic Using Interactive Multimedia for Pre-School Children. *Computer and Information Science*, 3 (4), 72–79. DOI: 10.5539/cis.v3n4p72
- Amaiwa, S. (2001). *The ripple effects and the future prospects of abacus learning*. College of Education, Shinshu University, Japan. Retrieved October 22, 2019. from www: https://www.shuzan.jp/english/brain/amaiwa.html.
- Amato, S., Hong, S. & Rosenblum, L. P. (2013). The abacus: Instruction by teachers of students with visual impairments. *Journal of Visual Impairment & Blindness*, 107 (4), 262–273. DOI: 10.1177/0145482X1310700403
- Anghileri, J. (2001). Intuitive approaches, mental strategies and standard algorithms. In: Anghileri, J. (Ed.). *Principles and practices in arithmetic teaching* (79-94). Buckingham: Open University Press.
- Baralis, G. H., Stavrou, I. G. & Mavrogianni, E. (2012). *A teaching approach using abacus applet for first grade children*. eRA 7- International Scientific Conference. Retrieved October 12, 2019. from www: https://era.teipir.gr/sites/default/files/c.3._education_iii_session.pdf.
- Banzić, D. (2017). *Mentalna aritmetika*. Beograd: Finesa.
- Bernazzani, D. (2005). *The Soroban/Abacus Handbook*. Retrieved October 12, 2019. from www: https://www.sliderulemuseum.com/Abaci/THE_ABACUS_HANDBOOK.pdf.
- Brawand, A. & Johnson, N. (2016). Effective methods for delivering mathematics instruction to students with visual impairments. *Journal of Blindness Innovation and Research*, 6 (1). Retrieved October 22, 2019. from www: https://www.nfb.org/images/nfb/publications/jbir/jbir16/jbir060101.html.

- Bhaskaran, M., Sengottaiyan, A., Madhu, S. & Ranganathan, V. (2006). Evaluation of memory in abacus learners. *Indian J Physiol Pharmacol*, 50, 225–233.
- Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E. & Empson, S. B. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 29 (1), 3–20. DOI: 10.2307/749715
- Chen, C. L., Wu, T. H., Cheng, M. C., Huang, Y. H., Sheu, C. Y., Hsieh, J. C. & Lee, J. S. (2006). Prospective demonstration of brain plasticity after intensive abacus-based mental calculation training: An fMRI study. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 569 (2), 567–571. DOI: 10.1016/j.nima.2006.08.101
- Dejić, M. & Mihajlović, A. (2015). Uloga i značaj istorije matematike u nastavi. *Godišnjak Učiteljskog fakulteta u Vranju*, VI, 67–82.
- Dejić, M. & Mihajlović, M. (2014). History of Mathematics and Teaching Mathematics. *Teaching Innovations*, 27 (3), 15–30. DOI: 10.5937/inovacije1403015D
- Freeman, N. (2014). Does the Japanese Abacus improve underachieving children's performance in mathematics? *The British Society for Research into Learning Mathematics*, 34 (3), 13–18. Retrieved October 22, 2019. from www: https://bsrlm.org.uk/wp-content/uploads/2016/02/BSRLM-IP-34-3-03.pdf.
- Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., Carpenter, T. P. & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 28 (2), 130–162. DOI: 10.2307/749759
- Gerhardt, C. J. (1858). *Leibnizens mathematische Schriften*. Halle: Band V. H. W. Schmidt.
- Hatano, G. & Osawa, K. (1983). Digit memory of grand experts in abacus-derived mental calculation. *Cognition*, 15 (1–3), 95–110. DOI: 10.1016/0010-0277(83)90035-5
- Heirdsfield, A. & Lamb, L. (2005). Mental Computation: The Benefits of Informed Teacher Instruction. In: Clarkson, P., Downtown, A., Gronn, D., Horne, M., McDonough, A., Pierce, R. & Roche, A. (Eds). *Proceedings MERGA 28 – 2005 Building connections: Theory, research and practice 2* (419–426). Melbourne. Retrieved October 12, 2019. from www: https://eprints.qut.edu.au/1889/1/1889.pdf.
- Heffelfinger, T. & Flom, G. (2004). *Abacus: Mystery of the bead.* Retrieved October 16, 2019. from www: https://totton.idirect.com/abacus/Abacus_Mystery_of_the_Bead.pdf.
- Hu, Y., Geng, F.,Tao, L., Hu, N., Du, F., & Fu, K. & Chen, F. (2011). Enhanced White Matter Tracts Integrity in Children With Abacus Training. *Human Brain Mapping*, 32 (1), 10–21. DOI: 10.1002/hbm.20996
- Huang, J., Du, F. L., Yao, Y., Wan, Q., Wang, X. S. & Chen, F. Y. (2015). Numerical magnitude processing in abacus-trained children with superior mathematical ability: An EEG study. *Journal of Zhejiang University-Science B*, 16 (8), 661–671. DOI: 10.1631/jzus.B1400287
- Jadhav, A. K. & Gathoo, V. S. (2018). Effect of Abacus Training on Numerical Ability of Students with Hearing Loss. *Disability, CBR & Inclusive Development*, 29 (2), 59–75.
- Kojima, T. (1954). *Advanced Abacus: Japanese Theory and Practice*. Tokyo: Charles. E. Tuttle Company.
- Lazić, B. (2014). Razvoj metodike nastave aritmetike od Srpske Preparandije do Učiteljskog fakulteta. U: Janković, P. i Stepanović, M. (ur.). *200 godina Srpske Preparandije u Sentandreji i Somboru* (325–336). Sombor: Pedagoški fakultet.

- Lee, Y., Lu, M. & Ko, H. (2007). Effects of skill training on working memory capacity. *Learning Instruction*, 17 (3), 336–344. DOI: 10.1016/j.learninstruc.2007.02.010
- Lean, C. B. & Lan, O. S. (2005). Comparing mathematical problem solving ability of pupils who learn abacus mental arithmetic and pupils who do not learn abacus mental arithmetic. Retrieved October 15, 2019. from www: https://ortk.pw/691.pdf.
- Lemonidis, C. (2015). *Mental Computation and Estimation: Implications for mathematics education research, teaching and learning*. London, United Kingdom: Taylor & Francis Ltd.
- Linsen, S., Verschaffel, L., Reynvoet, B. & De Smedt, B. (2015). The association between numerical magnitude processing and mental versus algorithmic multi-digit subtraction in children. *Learning and Instruction*, 35, 42–50. DOI: 10.1016/j.learninstruc.2014.09.003
- Maclellan, E. (2001). Mental Calculation: its place in the development of numeracy. *Westminster Studies in Education*, 24 (2), 145–154. DOI: 10.1080/0140672010240205
- Miller, K. F. & Stigler, J. W. (1991). Meanings of skill: Effects of Abacus Expertise on Number Representation. *Cognition and Instruction*, 8 (1), 29–67. DOI: 10.1207/s1532690xci0801_2
- Moeller, K., Pixner, S., Zuber, J., Kaufmann, L. & Nuerk, H. C. (2011). Early place-value understanding as a precursor for later arithmetic performance – A longitudinal study on numerical development. *Research in Developmental Disabilities*, 32 (5), 1837–1851. DOI:10.1016/j.ridd.2011.03.012
- Moon, P. (1971). *The abacus: its history, its design, its possibilities in the modern world*. New York: Gordon & Breach Science Publishers.
- Prentović, R., Sotirović, V. (1998). *Metodika razvoja početnih matematičkih pojmova*. Novi Sad: Didakta.
- Reys, B. J. & Barger, R. H. (1994). Mental computation: Issues from the United States perspective. In: Reys, R. E. & Nohda, N. (Eds.). *Computational alternatives for the twenty-first century* (31–47). Reston, Virginia: The National Council of Teachers of Mathematics.
- Rubenstein, R. N. (2001). Mental Mathematics beyond the Middle School: Why? What? How? *The Mathematics Teacher*, 94 (6), 442–446.
- Samoly, K. (2012). The history of abacus. *Ohio Journal of School Mathematics*, 65, 58–66.
- Sharma, M. C. (1993). Place value concept: How children learn it and how to teach it. *Math Notebook*, 3–25.
- Starkey, P. & Klein, A. (2008). Sociocultural influences on young children's mathematical knowledge. In: *Contemporary perspectives on mathematics in early childhood education* (253–276). Charlotte, NC: Information Age Publishing.
- Sarvari, E. R., Nasiri, H. & Abasi, P. (2015b). The Impact of Abacus on Mathematic Learning through Teachers' Innovative Behavior in Elementary Schools of Iran. *European Journal of Business and Management*, 7 (24), 125–130.
- Shanthala, B. N. (2011). *To study the effect of abacus learning on memory in school children* (doctoral dissertation). Karnataka, Bangalore: Rajiv Gandhi University of Health Sciences. Retrieved October 16, 2019. from www: http://52.172.27.147:8080/jspui/bitstream/123456789/6995/1/Shanthala%20B%20N.pdf.
- Stigler, J., Chalip, L. & Miller, K. (1986). Consequences of Skill: The Case of Abacus Training in Taiwan. *American Journal of Education*, 94 (4), 447–479.

- Shwalb, D., Sugie, S. & Yang, C. (2004). Motivation for Abacus Studies and School Mathematics a Longitudinal Study of Japanese 3rd–6th Graders. In: Shwalb, D., Nakazawa, J. & Shwalb, B. J. (Eds.). *Applied Developmental Psychology: Theory, Practice, and Research from Japan* (109–135). Information Age Publishing Inc.
- Škorvaga, S. (2011). *Računska ograničenja kineskog abakusa*. Osijek: Sveučilište J. J. Strossmayera u Osijeku.
- Špijunović, K. & Maričić, S. (2016). *Metodika početne nastave matematike*. Užice: Učiteljski fakultet.
- Tan, P. H., Moody, L., Bromage, A. & Richards, C. (2009). *Cross-cultural transfer of the abacus for teaching mathematics. The Asian Conference on Education 2009* (1007–1018). Japan: Osaka. Retrieved October 12. 2019. from www: https://www.academia.edu/200035/CrossCultural_Transfer_of_the_Abacus_for_Teaching_Mathematics.
- Tanaka, S., Seki, K., Hanakawa, T., Harada, M., Sugawara, S. K., Sadato, N., Watanabe, K. & Honda, M. (2012). Abacus in the brain: a longitudinal functional MRI study of a skilled abacus user with a right hemispheric lesion. *Frontiers in psychology*, 3 (315), DOI: 10.3389/fpsyg.2012.00315
- Tiwari, M., Tiwari, Y. & Patil, P. (2017). Effective impact of whole brain development among the abacus learners of younger generation. *International Journal of Multidisciplinary Research and Development*, 4 (12), 61–65.
- Threlfall, J. (2009). Strategies and flexibility in mental calculation. *ZDM Mathematics Education*, 41, 541–555. DOI: 10.1007/s11858-009-0195-3
- Van den Heuvel-Panhuizen, M. (Ed.) (2001). *Children learn mathematics. A teaching-learning trajectory with intermediate attainment targets for calculation with whole numbers in primary school.* Utrecht, The Netherlands: Freudenthal Institute. University of Utrecht. Retrieved January 18, 2017 from www: https://www.sensepublishers.com/media/161-children-learn-mathematics.pdf.
- Vasuki, K. (2013). *The impact of abacus learning of mental arithmetic on cognitive abilities of children. Aloha Mental Arithmetic*. Retrieved September 29, 2019. from www: https://www.alohaspain.com/public/file/ALOHA_Benefits_ScientificReport.pdf.
- Verschaffel, L., Greer, B. & De Corte, E. (2007). Whole number concepts and operations. In: Lester, F. K. (Ed.). *Second handbook of research on mathematics teaching and learning* (557–628). Greenwich, CT: Information Age Publishing.
- Vita, A. C. & Kataoka, V. Y. (2014). Blind students' learning of probability through the use of a tactile model. *Statistics Education Research Journal*, 13 (2), 148–163. Retrieved November 1, 2019. from www: http://iase-web.org/documents/SERJ/SERJ13%282%29_Vita.pdf.
- Wang, C., Geng, F., Yao, Y., Weng, J., Hu, Y. & Chen, F. (2015). Abacus Training Affects Math and Task Switching Abilities and Modulates Their Relationships in Chinese Children. *PLoS ONE*, 10 (10), 1–15, DOI: 10.1371/journal.pone.0139930
- Zeljić, M. Ž., Ilić, S. M. & Jelić, M. S. (2017). Mentalna aritmetika – strategije oduzimanja. *Inovacije u nastavi*, 30 (4), 49–61. DOI: 10.5937/inovacije1704049Z

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АБАКУС РАЧУНАЉКА – ОД ИСТОРИЈЕ ДО ПРИМЕНЕ У МАТЕМАТИЧКОМ ОБРАЗОВАЊУ

Имајући у виду да историја математике може имати значајну улогу у математичком образовању данас, у раду скрећемо пажњу на древну рачунаљку – абакус. Абакус је најстарија позната алатка за рачунање која је коришћена у Месопотамији, старом Египту, античкој Грчкој и Риму, а унапређени модел абакуса у употреби је и у данашње време. У раду представљамо различите верзије абакуса које су се користиле током историје (грчки, римски, кинески, јапански, руски и индијски абакус (механички и дигитални), а посебно се бавимо истраживањем утицаја јапанског абакуса (Соробан) на способност рачунања, будући да се овај модел рачунаљке сматра најефикаснијим, јер се базира искључиво на децималном нумеричком систему. Циљ овог рада је да применом методе теоријске анализе истражимо вредност, могућности и улогу примене јапанског Соробана у раду са децом. Посебно скрећемо пажњу на употребу абакуса у математичком образовању кроз следеће аспекте: у учењу менталне аритметике, у схватању месне вредности цифре, у менталном развоју деце, у раду са децом са сметњама у развоју, нарочито са децом са оштећењем вида или слуха, као и погледу сврсисходности абакуса као средства за развијање вештина за решавање проблема.

Сматрамо да методолошки приступ који треба да доведе до разумевања појма броја мора да буде заснован на конкретним, актуелним и очигледним факторима који омогућавају јасно формулисање појма броја и вредности сваке цифре, за шта абакус може бити добра основа. Употребом абакуса ученици упознају месне вредности сваке цифре у броју, а тиме стичу дубље разумевање структуре броја уз помоћ куглица на рачунаљки, јер им физичка и не-симболичка представа броја на абакусу помаже да визуализују положај места цифре, што доприноси бољем разумевању структуре броја. Ученици се обучавају да користе абакус за рачунање превенствено кроз конкретне активности кроз које уче како да померају куглице на абакусу, чиме се подстичу да стварају менталне слике и тако боље развијају менталне стратегије које ће им омогућити лакши прелаз са конкретних активности на ментални ниво рачунања. Кроз овај процес деца боље разумеју математику и могу да решавају проблеме и примењују математику, постају флексибилнија у решавању математичких проблема, што су све предности менталног рачунања. Применом абакуса у рачунању све активности учења прелазе са нивоа практичног-перцептивног резоновања на ниво концептуално-логичког резоновања, зато што оне од самог почетка пружају конкретну визуелну и манипулативну основу за учење, што се директно одражава на

ментални развој деце. Тренутно успостављање везе између покрета руке и рада мозга доприноси бржем и уравнотеженом укупном развоју мозга. Ментално рачунање подстиче менталну активност, концентрацију, меморију, аналитичке вештине и стрпљење који се развијају применом абакуса у раду са децом, док визуелизација доприноси развоју неурона у мозгу. Поред тога, корист од употребе овог средства за рачунање нарочито је видљива у раду са децом са оштећеним слухом или видом, зато што им абакус олакшава да визуелизују ментално рачунање кроз кинестетичку активност. Померањем куглица на рачунаљки ова деца могу да визуелизују одређене активности у оквиру теме о бројевима, да их пренесу на ментални ниво и да развију вештину менталног рачунања, чиме се превазилазе тешкоће које имају у вези са језиком, било говорним или писаним.

На основу бројних истраживачких радова и ставова аутори овог рада закључују да јапански абакус у раду са децом у оквиру математичког образовања може остварити бројне добробити, међу којима су свакако најзначајнији појмовно разумевање менталне аритметике и развијање способности менталног рачунања и решавања проблема, стицање јасне менталне представе о структури бројева, њиховој величини и односу, разумевање месне вредности цифре, али и развијање мотивације и позитивних ставова према математици.

У раду се бавимо само неким аспектима примене јапанског абакуса у раду са децом, са циљем да скренемо пажњу на ово древно средство за рачунање и на изузетну корист коју од њега можемо имати у учењу математике.

Кључне речи: *абакус, историја математике, соробан, ментално рачунање, математичко образовање.*