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The Correlation between Figural and Conceptual Properties of Angle and Cube in Pre-Service Teachers Geometric Reasoning[2]

Extended summary

Drawing upon Fischbein's theory of figural concepts, the starting point of the paper is the use and value of the history of geometry in mathematics education - first point that we make historical reference are theorems of Eudemus of Rhodes and Thales of Miletus and the second one is elaboration of these theorems in work of Serbian mathematicians Mihailo Petrović Alas. Fischbein's theory is mainly based on a hypothesis that geometry deals with mental entities, the so-called geometric figures, which simultaneously possess conceptual (e.g. ideality, abstractness, generality) and figural properties (e.g. shape, position, magnitude). In mathematics, the definition of the objects is created directly or by deduction, therefore, the interpretation of the figural components of geometric figures should entirely be subjected to formal constraints. However, Vinner and Hershkowitz (in Dreyfus, 2014: 50) pointed out that student's geometric reasoning may not be based on definition, but on prototype examples. In the paper, we explored different examples from the history of geometry in mathematics education involving the situation in which an image or non-prototype example creates a conflict that draws upon the real conceptual understanding of an object, i.e., the image of an object dominates the formal definition. The concept image (Vinner, Hershkowitz, Tall, in Dreyfus, 2014: 51), defined as a total cognitive structure associated with a concept in an individual's mind (e.g. pictures, examples, words) can deviate from the formal definition of the concept (Dreyfus, 2014). The integration

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of conceptual and figural properties, with the predominance of conceptual constraints over the figural ones, according to Fischbein, is not a natural process. Vinner, Hershkowitz and Tall hold that in mathematics education the cognitive processes by which concepts are conceived must be distinguished from the mathematical concepts as formally defined. This process requires teachers' continual, systematic, and committed involvement. Conflict situations should serve to teach students how to follow carefully the requirements set in definitions, which sometimes can run contrary to what the images seem to represent (or impose).

The goal of the research was to examine the pre-service primary school teachers' geometric reasoning regarding the correlation between figural (pictorial) and conceptual properties of geometric objects. We have formulated two research objectives: 1) to investigate whether the figural structure of an angle dominates in the geometric reasoning of the pre-service primary school teachers over their formal conceptual constraints and 2) to investigate in which situations an image can help the pre-service primary school teachers reasoning when there is a domination of one of the two above-mentioned aspects over the other. The non-random convenience research sample consisted of 64 fourth-year students of the Teacher Education Faculty in Belgrade, Serbia (pre-service teachers). A descriptive method and testing as a technique were used in the research. The obtained data were analysed quantitatively and qualitatively. The research procedure involved four tasks: The first task involved defining an angle and determining which of the points on the picture appertain to the angle on the prototypical picture and on the picture where angle was a part of the rectangle. The second task was to determine whether the lines (straight line, semi-straight line and segment line) had a common point. The third and fourth tasks involved a net of a cube and imagining what the net would look like if it were folded in a cube. The last two tasks were repeated with an additional instruction: students had to sketch and mark the folded net of a cube.

The results of the study show that the figural (pictorial) structure of an angle dominates in the geometric reasoning of the pre-service primary school teachers over its formal conceptual constraints. Even though students could define the angle and understand different types of lines, in observing the image they failed to show the understanding of the concept of an angle. Also, students were significantly more successful at identifying the points appertaining to the prototypical image of an angle than at identifying the points appertaining to an angle of a polygon.

Considering the second research objective, there were some differences in the situations when an image is helpful in solving tasks involving the concepts of the net of the cube and the cube itself. On the task that was more difficult (students had to consider overlapping sides and vertices) an image did not improve students' achievement on the task, while in an easier example (that refers to the faces of the cube), the results show a significant change in the students' achievement, which implies that such tasks are an opportunity for developing the pre-service primary school teachers' reasoning, whereas an image can be useful only in specific and less complex situations.

We conclude that it is necessary to work on figural (pictorial) and conceptual aspects of geometric objects to resolve conflict situations and strengthen the understanding of the correlation between figural and conceptual properties in the initial geometry instruction. Also, the

primary school teachers' knowledge impacts the design of instructions and it is important for them to know the key attributes of geometric objects, understand definitions and the role of definitions in mathematics, as well as to know how to select the proper examples that impact the planning of activities involving these objects.

Keywords: history of geometry, figural concept, geometric definition, instruction, geometric reasoning.

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