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The Algebraization of Mathematics: Using Original Sources for Learning Mathematics[2]

Extended summary[3]

The knowledge of the processes in the history of mathematics is very useful for a better comprehension of the foundations and nature of mathematics. The activities, based on the analysis of significant historical sources, contribute to improving mathematicians' integral formation, giving them additional knowledge of the social and scientific context of the periods involved, while mathematics can be considered as an intellectual activity for solving problems in each period. History shows that societies develop because of the scientific activity and that mathematics is a fundamental part of this process. Reading ancient texts enables us to develop a vision of mathematics not as a final and finished product, but as a useful, dynamic, human, interdisciplinary and heuristic science, which has developed through efforts to solve the problems about the world around us that humanity has been faced with throughout history.

The aim of this paper, within the framework of the process of algebraization of mathematics, is to analyze the results of the implementation of a historical activity in a mathematics history course for the bachelor's degree of mathematics. The activity involves using singular geometric constructions for solving equations.

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The algebraization of mathematics was a key process in the transformation of the seventeenth-century mathematics if we consider two of its relevant features: a new symbolic language and an analytic method for solving problems. In fact, the publication in 1591 of *In Artem Analyticen Isagoge* by François Viète (1540-1603) constituted a step forward in this process with the development of the symbolic language and the analytic method for solving problems. Later, Pierre de Fermat (1601-1665) was among the mathematicians who used this algebraic analysis to solve geometric problems. However, the most influential figure in this process of algebraization was René Descartes (1596-1650) who published *La Géométrie* in 1637 as an appendix to the *Discours de la Méthode*. In fact, the study of the origins of polynomials and their associated equations provides a history of geometrical constructions for the solution of equations with instructive and informative passages for students, whether college degree or high school students.

In this paper we present an activity, set within the framework of the algebraization of the seventeenth-century mathematics, which contains singular geometric constructions used for solving equations. The activity was implemented in the course for the bachelor's degree in mathematics. We describe and compare the geometrical justifications developed by Viète and Descartes in their works using original sources, and then we analyze the students' refelections on the meaning of the algebraization of mathematics in providing answers to some questions prepared by the teacher.

The paper shows that students gain a better understanding of the role played by the relationship between algebra and geometry in the development of mathematics, which helps them to improve their mathematical training. The most significant results have been obtained from students' comments. Making and comparing these geometrical constructions also helps the students to develop the analytic and synthetic reasoning and improve their mathematics knowledge. In addition, the analysis and the reflections of students on these historical geometrical constructions using algebraic expressions provide us with a rich source of ideas regarding the relationship between algebra and geometry throughout history. Through this activity students learn that at the end of the process of the algebraization of mathematics, algebra and geometry became complementary and that it was from the coordination and conjunction of both branches that new fields of mathematics developed in the path to modern mathematics.

Keywords: seventeenth century, learning mathematics, algebra, geometry, algebraization of mathematics.

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