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Abacus computing tool – from history to application in mathematical education[2]

Extended summary

The history of mathematics plays a significant role in mathematics education. The paper draws attention to the ancient calculating tool – abacus – in the context of analyzing the role of mathematics history in mathematics education. The abacus is the oldest known manual calculating instrument in history, used in ancient Mesopotamia, Egypt, Greece and Rome, whereas an advanced model of the abacus is still in use. The paper presents different versions of the abacus that emerged throughout history (Salamis, Roman, Chinese, Japanese, Russian, and Indian abacus (mechanical and digital)), with a special emphasis on exploring the importance of the Japanese abacus (Soroban) which is considered the most efficient version, given it is adapted solely to the decimal numeral system. The paper aims at exploring the role, values and possibilities of applying the Soroban in mathematics education of children through the method of theoretical analysis. We particularly focused on the application of the abacus on the following aspects of mathematics education: learning mental arithmetic, understanding the local value of a digit, mental development of children, treatment of children with developmental disabilities, primarily the children with visual and hearing impairments, and its contribution as a means that stimulates problem-solving skills.

Authors argue that a methodological approach which should lead to the conceptual understanding of the concept of number must be based on concrete, actual, and obvious fac-

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tors that result in the formation of a clear concept of number and value of each digit in that number, and that the abacus provide a good basis for that. Using the abacus, students are introduced to the local value of each digit in a number, thus gaining a deeper understanding of the number structure with the help of beads, where a physical and non-symbolic representation of a number on the abacus helps them visualize the position of the local value of a digit, which contributes to the overall understanding of the structure of numbers. Students are trained to use the abacus for calculus, primarily through concrete activities of moving the beads, an activity that creates mental representations, thus stimulating the development of mental strategies which allow the transition from concrete activities to the mental plane of calculus. This way, children gain a deeper understanding of mathematics and are able to solve problems, apply mathematics, and acquire flexibility and the ability of adaption in problem solving, all of which are the advantages of mental calculation. Through the application of the abacus in calculation, all learning activities are transferred from the level of practical-perceptual thinking to the level of conceptual-logical thinking, because these activities create a concrete visual and manipulative basis for learning from the very beginning, which directly reflects on mental development. The fast communication occurring between one's hands and brain helps a faster and balanced overall development of the brain. Mental calculation alone stimulates mental activity, concentration, memory, analytical skills and patience that develop through practice and work on the abacus, whereas visualization contributes to the development of neurons in the brain. In addition, the remarkable usefulness of this tool is reflected in its application in teaching mathematics to children with visual and hearing impairments because, in these situations, it helps the visualization of mental calculation through a kinesthetic action. Through the manipulation of the beads on the abacus, these children are able to visualize specific activities with numbers, transfer them to the mental plane and develop the skills of mental calculus which compensate for their language deficiencies, whether spoken or written.

Based on numerous studies and views, authors can conclude that the Japanese abacus can provide various benefits in mathematics education of children, the most important of which are certainly conceptual understanding of mental arithmetic, developing the skills of mental calculation and problem solving, acquiring a clear mental image of the structure of numbers, their value and relations, understanding the local value of a digit, and finally, developing motivation and positive attitude toward mathematics. The paper underlines only some aspects of the application of the Japanese abacus in teaching children, all of which are aimed at drawing attention to this ancient calculating tool, and its exceptional usefulness in mathematics education.

Keywords: abacus, history of mathematics, soroban, mental computation, mathematical education.

References

- Ahamed, B. (2012). *Abacus: The next generation. Brain Education & Training Skils*. Retrieved September 22, 2019. from www: https://www.indianabacus.com.

- Ahmad, S. Z., Rosmani, A. F., Ismail, M. H. & Shakeri, M. (2010). An Introductory of Mental Arithmetic Using Interactive Multimedia for Pre-School Children. *Computer and Information Science*, 3 (4), 72–79. DOI: 10.5539/cis.v3n4p72
- Amaiwa, S. (2001). *The ripple effects and the future prospects of abacus learning*. College of Education, Shinshu University, Japan. Retrieved October 22, 2019. from www: https://www.shuzan.jp/english/brain/amaiwa.html.
- Amato, S., Hong, S. & Rosenblum, L. P. (2013). The abacus: Instruction by teachers of students with visual impairments. *Journal of Visual Impairment & Blindness*, 107 (4), 262–273. DOI: 10.1177/0145482X1310700403
- Anghileri, J. (2001). Intuitive approaches, mental strategies and standard algorithms. In: Anghileri, J. (Ed.). *Principles and practices in arithmetic teaching* (79-94). Buckingham: Open University Press.
- Baralis, G. H., Stavrou, I. G. & Mavrogianni, E. (2012). *A teaching approach using abacus applet for first grade children*. eRA 7- International Scientific Conference. Retrieved October 12, 2019. from www: https://era.teipir.gr/sites/default/files/c.3._education_iii_session.pdf.
- Banzić, D. (2017). *Mentalna aritmetika*. Beograd: Finesa.
- Bernazzani, D. (2005). *The Soroban/Abacus Handbook*. Retrieved October 12, 2019. from www: https://www.sliderulemuseum.com/Abaci/THE_ABACUS_HANDBOOK.pdf.
- Brawand, A. & Johnson, N. (2016). Effective methods for delivering mathematics instruction to students with visual impairments. *Journal of Blindness Innovation and Research*, 6 (1). Retrieved October 22, 2019. from www: https://www.nfb.org/images/nfb/publications/jbir/jbir16/jbir060101.html.
- Bhaskaran, M., Sengottaiyan, A., Madhu, S. & Ranganathan, V. (2006). Evaluation of memory in abacus learners. *Indian J Physiol Pharmacol*, 50, 225-233.
- Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E. & Empson, S. B. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 29 (1), 3-20. DOI: 10.2307/749715
- Chen, C. L., Wu, T. H., Cheng, M. C., Huang, Y. H., Sheu, C. Y., Hsieh, J. C. & Lee, J. S. (2006). Prospective demonstration of brain plasticity after intensive abacus-based mental calculation training: An fMRI study. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 569 (2), 567–571. DOI: 10.1016/j.nima.2006.08.101
- Dejić, M. & Mihajlović, A. (2015). Uloga i značaj istorije matematike u nastavi. *Godišnjak Učiteljskog fakulteta u Vranju*, VI, 67-82.
- Dejić, M. & Mihajlović, M. (2014). History of Mathematics and Teaching Mathematics. *Teaching Innovations*, 27 (3), 15-30. DOI: 10.5937/inovacije1403015D
- Freeman, N. (2014). Does the Japanese Abacus improve underachieving children's performance in mathematics? *The British Society for Research into Learning Mathematics*, 34 (3), 13-18. Retrieved October 22, 2019. from www: https://bsrlm.org.uk/wp-content/uploads/2016/02/BSRLM-IP-34-3-03.pdf.

- Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., Carpenter, T. P. & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 28 (2), 130–162. DOI: 10.2307/749759
- Gerhardt, C. J. (1858). *Leibnizens mathematische Schriften*. Halle: Band V. H. W. Schmidt.
- Hatano, G. & Osawa, K. (1983). Digit memory of grand experts in abacus-derived mental calculation. *Cognition*, 15 (1–3), 95–110. DOI: 10.1016/0010-0277(83)90035-5
- Heirdsfield, A. & Lamb, L. (2005). Mental Computation: The Benefits of Informed Teacher Instruction. In: Clarkson, P., Downtown, A., Gronn, D., Horne, M., McDonough, A., Pierce, R. & Roche, A. (Eds). *Proceedings MERGA 28 – 2005 Building connections: Theory, research and practice 2* (419–426). Melbourne. Retrieved October 12, 2019. from www: https://eprints.qut.edu.au/1889/1/1889.pdf.
- Heffelfinger, T. & Flom, G. (2004). *Abacus: Mystery of the bead.* Retrieved October 16, 2019. from www: https://totton.idirect.com/abacus/Abacus_Mystery_of_the_Bead.pdf.
- Hu, Y., Geng, F.,Tao, L., Hu, N., Du, F., & Fu, K. & Chen, F. (2011). Enhanced White Matter Tracts Integrity in Children With Abacus Training. *Human Brain Mapping*, 32 (1), 10–21. DOI: 10.1002/hbm.20996
- Huang, J., Du, F. L., Yao, Y., Wan, Q., Wang, X. S. & Chen, F. Y. (2015). Numerical magnitude processing in abacus-trained children with superior mathematical ability: An EEG study. *Journal of Zhejiang University-Science B*, 16 (8), 661–671. DOI: 10.1631/jzus.B1400287
- Jadhav, A. K. & Gathoo, V. S. (2018). Effect of Abacus Training on Numerical Ability of Students with Hearing Loss. *Disability, CBR & Inclusive Development*, 29 (2), 59–75.
- Kojima, T. (1954). *Advanced Abacus: Japanese Theory and Practice*. Tokyo: Charles. E. Tuttle Company.
- Lazić, B. (2014). Razvoj metodike nastave aritmetike od Srpske Preparandije do Učiteljskog fakulteta. U: Janković, P. i Stepanović, M. (ur.). *200 godina Srpske Preparandije u Sentandreji i Somboru* (325–336). Sombor: Pedagoški fakultet.
- Lee, Y., Lu, M. & Ko, H. (2007). Effects of skill training on working memory capacity. *Learning Instruction*, 17 (3), 336–344. DOI: 10.1016/j.learninstruc.2007.02.010
- Lean, C. B. & Lan, O. S. (2005). Comparing mathematical problem solving ability of pupils who learn abacus mental arithmetic and pupils who do not learn abacus mental arithmetic. Retrieved October 15, 2019. from www: https://ortk.pw/691.pdf.
- Lemonidis, C. (2015). *Mental Computation and Estimation: Implications for mathematics education research, teaching and learning*. London, United Kingdom: Taylor & Francis Ltd.
- Linsen, S., Verschaffel, L., Reynvoet, B. & De Smedt, B. (2015). The association between numerical magnitude processing and mental versus algorithmic multi-digit subtraction in children. *Learning and Instruction*, 35, 42–50. DOI: 10.1016/j.learninstruc.2014.09.003
- Maclellan, E. (2001). Mental Calculation: its place in the development of numeracy. *Westminster Studies in Education*, 24 (2), 145–154. DOI: 10.1080/0140672010240205

- Miller, K. F. & Stigler, J. W. (1991). Meanings of skill: Effects of Abacus Expertise on Number Representation. *Cognition and Instruction*, 8 (1), 29-67. DOI: 10.1207/s1532690xci0801_2
- Moeller, K., Pixner, S., Zuber, J., Kaufmann, L. & Nuerk, H. C. (2011). Early place-value understanding as a precursor for later arithmetic performance – A longitudinal study on numerical development. *Research in Developmental Disabilities*, 32 (5), 1837-1851. DOI:10.1016/j.ridd.2011.03.012
- Moon, P. (1971). *The abacus: its history, its design, its possibilities in the modern world*. New York: Gordon & Breach Science Publishers.
- Prentović, R., Sotirović, V. (1998). *Metodika razvoja početnih matematičkih pojmova*. Novi Sad: Didakta.
- Reys, B. J. & Barger, R. H. (1994). Mental computation: Issues from the United States perspective. In: Reys, R. E. & Nohda, N. (Eds.). *Computational alternatives for the twenty-first century* (31-47). Reston, Virginia: The National Council of Teachers of Mathematics.
- Rubenstein, R. N. (2001). Mental Mathematics beyond the Middle School: Why? What? How? *The Mathematics Teacher*, 94 (6), 442-446.
- Samoly, K. (2012). The history of abacus. *Ohio Journal of School Mathematics*, 65, 58-66.
- Sharma, M. C. (1993). Place value concept: How children learn it and how to teach it. *Math Notebook*, 3-25.
- Starkey, P. & Klein, A. (2008). Sociocultural influences on young children's mathematical knowledge. In: *Contemporary perspectives on mathematics in early childhood education* (253-276). Charlotte, NC: Information Age Publishing.
- Sarvari, E. R., Nasiri, H. & Abasi, P. (2015b). The Impact of Abacus on Mathematic Learning through Teachers' Innovative Behavior in Elementary Schools of Iran. *European Journal of Business and Management*, 7 (24), 125-130.
- Shanthala, B. N. (2011). *To study the effect of abacus learning on memory in school children* (doctoral dissertation). Karnataka, Bangalore: Rajiv Gandhi University of Health Sciences. Retrieved October 16, 2019. from www: http://52.172.27.147:8080/jspui/bitstream/123456789/6995/1/Shanthala%20B%20N.pdf.
- Stigler, J., Chalip, L. & Miller, K. (1986). Consequences of Skill: The Case of Abacus Training in Taiwan. *American Journal of Education*, 94 (4), 447-479.
- Shwalb, D., Sugie, S. & Yang, C. (2004). Motivation for Abacus Studies and School Mathematics a Longitudinal Study of Japanese 3rd-6th Graders. In: Shwalb, D., Nakazawa, J. & Shwalb, B. J. (Eds.). *Applied Developmental Psychology: Theory, Practice, and Research from Japan* (109-135). Information Age Publishing Inc.
- Škorvaga, S. (2011). *Računska ograničenja kineskog abakusa*. Osijek: Sveučilište J. J. Strossmayera u Osijeku.
- Špijunović, K. & Maričić, S. (2016). *Metodika početne nastave matematike*. Užice: Učiteljski fakultet.
- Tan, P. H., Moody, L., Bromage, A. & Richards, C. (2009). *Cross-cultural transfer of the abacus for teaching mathematics. The Asian Conference on Education 2009* (1007–1018). Japan: Osaka.

Retrieved October 12. 2019. from www: https://www.academia.edu/200035/CrossCultural_Transfer_of_the_Abacus_for_Teaching_Mathematics.

- Tanaka, S., Seki, K., Hanakawa, T., Harada, M., Sugawara, S. K., Sadato, N., Watanabe, K. & Honda, M. (2012). Abacus in the brain: a longitudinal functional MRI study of a skilled abacus user with a right hemispheric lesion. *Frontiers in psychology*, 3 (315), DOI: 10.3389/fpsyg.2012.00315
- Tiwari, M., Tiwari, Y. & Patil, P. (2017). Effective impact of whole brain development among the abacus learners of younger generation. *International Journal of Multidisciplinary Research and Development*, 4 (12), 61-65.
- Threlfall, J. (2009). Strategies and flexibility in mental calculation. *ZDM Mathematics Education*, 41, 541-555. DOI: 10.1007/s11858-009-0195-3
- Van den Heuvel-Panhuizen, M. (Ed.) (2001). *Children learn mathematics. A teaching-learning trajectory with intermediate attainment targets for calculation with whole numbers in primary school.* Utrecht, The Netherlands: Freudenthal Institute. University of Utrecht. Retrieved January 18, 2017 from www: https://www.sensepublishers.com/media/161-children-learn-mathematics.pdf.
- Vasuki, K. (2013). *The impact of abacus learning of mental arithmetic on cognitive abilities of children. Aloha Mental Arithmetic*. Retrieved September 29, 2019. from www: https://www.alohaspain.com/public/file/ALOHA_Benefits_ScientificReport.pdf.
- Verschaffel, L., Greer, B. & De Corte, E. (2007). Whole number concepts and operations. In: Lester, F. K. (Ed.). *Second handbook of research on mathematics teaching and learning* (557–628). Greenwich, CT: Information Age Publishing.
- Vita, A. C. & Kataoka, V. Y. (2014). Blind students' learning of probability through the use of a tactile model. *Statistics Education Research Journal*, 13 (2), 148-163. Retrieved November 1, 2019. from www: http://iase-web.org/documents/SERJ/SERJ13%282%29_Vita.pdf.
- Wang, C., Geng, F., Yao, Y., Weng, J., Hu, Y. & Chen, F. (2015). Abacus Training Affects Math and Task Switching Abilities and Modulates Their Relationships in Chinese Children. *PLoS ONE*, 10 (10), 1-15, DOI: 10.1371/journal.pone.0139930
- Zeljić, M. Ž., Ilić, S. M. & Jelić, M. S. (2017). Mentalna aritmetika – strategije oduzimanja. *Inovacije u nastavi*, 30 (4), 49-61. DOI: 10.5937/inovacije1704049Z