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**Short
scientific paper**

Use of Abu'l-Wafa's problem to foster pedagogical knowledge of prospective mathematics teachers

Extended summary

Mathematics teachers have to master mathematical contents, as well as pedagogical content knowledge that will enable them to create an environment in which pupils will learn in an optimal and effective way. In mathematics teaching and learning it is equally important to develop both procedural and conceptual knowledge among pupils. Mathematics teachers need to find a "right measure" in teaching, paying attention both to performing mathematical procedures and giving explanations, argumentations and proof (Pjanić, 2019). They have to promote such mathematical culture among their pupils. Using episodes from the history of mathematics could help teachers to accomplish this.

Solving Abu'l-Wafa's problem could be a powerful tool for building and fostering pedagogical content knowledge of prospective mathematics teachers. Abu'l-Wafa in his treatise On Those Parts of Geometry Needed by Craftsmen described several constructions made with the aid of ruler and "rusty compass", a compass with a fixed angle. These included constructing a perpendicular at the endpoint of a line segment, dividing segments in equal parts, bisecting angles, constructing a square in a circle and, constructing a regular pentagon (Berggren, 2003).

The goal of this case study is to examine if this episode in the history of mathematics would foster the subject content knowledge and pedagogical content knowledge of a group of prospective mathematics teachers.

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Abu'l-Wafa's problem was presented to five prospective mathematics teachers attending the Mathematics Teaching Methodology Course. The phases of the experiment were:

- Discussion about learning trajectory that includes the notions of bisector, perpendicular, angle and circle.
- Presentation of an episode from the history of mathematics: Abu'l Wafa's problem.
- Homework: Students had to analyse the mathematics programs for elementary and high school and to look at some textbooks commonly used at these levels.
- Homework: Students had to solve the problem and to design the teaching sequence/ didactic situation that includes the problem. The sequences designed by the prospective teachers had to fulfill the program requirements and be suitable for the relevant school level.
- Discussion among students and confrontation of the produced learning sequences / didactic situations. In this phase the focus is on linking different mathematics ideas and stressing the possibilities of a rich fundus of mathematics history that can be used in the process of teaching mathematics.

They had to find out at what point of learning geometry and how to engage pupils to solve this problem, taking into account the question of procedural and conceptual knowledge in mathematics as well as the important question of the role of proof and argumentation in mathematics classes.

The participants provided five different solutions to the given problem: construction of right angle at vertex A, construction of bisector of AB and translation to the endpoint A, Abu'I Wafa's solution – link to central and peripheral angle, construction of regular hexagon, proposal and solution of the auxiliary problem that led to the solution of Abu'l Wafa's problem.

During the discussion phase, the students agreed that a learning situation at high-school level could be created in such a manner that a teacher presents the Abu'l Wafa's problem and asks pupils to solve it. The students assumed that high school pupils could not link the solution to the problem to central and peripheral angles by themselves. Accordingly, they suggested that teachers should present Abu'l Wafa's construction and ask pupils to prove it.

Our case study showed that an integration of the history of mathematics in education may be particularly relevant for supporting and improving the pedagogical content knowledge of prospective mathematics teachers. The prospective mathematics teachers who participated in the study successfully perceived different connections between concepts, provided multiple solutions and proofs, discussed different aspects of the problem's conditions, and connected them with possible solutions. Finally, they created didactic situations including the given problem.

Keywords: Abu'l-Wafa's problem, mathematical and pedagogical content knowledge, prospective mathematics teachers.

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