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Second-Graders' Understanding of Comparison Tasks[2]

Extended summary

Textual tasks are considered the basis for learning mathematics, so extensive research is focused on the analysis of various problems and obstacles faced by students in solving these tasks. The classification of the single mathematical operation tasks into combining, modification, and comparison tasks has been used as a starting point in many studies. The focus of our research is on comparison tasks. Although all three subtypes of comparison problems (unknown difference set, compared or reference set) describe a comparative relationship, students struggle the most with the tasks with unknown reference sets, followed by tasks with unknown comparison sets, while the tasks with an unknown difference between sets are the easiest for students. Opinions differ about the source of students' difficulties with comparison problems. In the published literature, the main cause of the difficulties that students have in solving comparison tasks is the consistency effect, which is reflected in the use of the wrong operation due to a lack of understanding of relational terminology. Another possible explanation for the difficulties with comparison problems many researchers find in the answer to the question of how explicitly the semantic relations between the given and unknown quantities of the problem are expressed. The third and most frequently researched hypothesis that could explain the difficulties in solving the comparison tasks refers to the approach to solving the problem which

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is called differently in the literature: “Compute first and think later”, “key word” and “number grabbing”.

The research is a part of a broader study in which we deal with the achievements of the 2nd, 4th and 6th grade students on combination-comparison tasks. This paper presents the results obtained in the second grade. The aim of this research is to examine the second-graders’ understanding of relational terminology and the application of that understanding to comparison tasks of varying structure and complexity. The elements that affect the structure and complexity of such problems are: 1) number of comparisons in a task - one or two comparisons 2) the ratio of the compared and reference sets in tasks with two comparisons 3) consistency of language in a task.

The results of the research show that second-grade students, who made up our sample, did statistically significantly better on the tasks with consistent language formulation (CL) than the tasks with inconsistent language formulation (IL) in each of the semantic groups of tasks. The existence of a moderate connection between KJ and NJ formulations was obtained only on the tasks with a more complex structure. These results suggest that it is only on the tasks with a more complex structure that it is possible to examine whether students demonstrate a deeper understanding of the problem of comparison.

The number of comparisons in the task, as well as the different semantic structure of the problem, did not prove to be significant factors affecting student success. There is no significant difference in performance, and there is a strong correlation in achievement on these tasks. On the tasks of inconsistent wording, errors occurred due to misunderstanding of the relational term, i.e. due to the consistency effect. On the other hand, in the tasks where the sequence of the data cannot be directly translated into a mathematical expression using the keyword approach, although the KJ formulation was used, students made errors that show a misunderstanding of the structure of the task. Therefore, the difficulties that arise in solving textual comparison tasks are not only due to the wrongly chosen operation, but also due to the inability to thoroughly understand and present the text of the problem. To solve textual tasks, many students used the “abbreviated” model, where they immediately translated the text of the problem into a mathematical model, relying on a quick and superficial analysis of the problem, that is, on the key words in the text. After receiving the results, the students did not consider the text of the problem nor did they check whether the solution corresponds to the original problem situation.

Our results show that in teaching mathematics, it is important to treat the relations it is greater than/less than as the relations of strict order, because the first step towards a conceptual understanding of the problem of comparison is a two-way reading of these relations. The next steps should include the use of tasks of different semantic structures and complexity that contribute to the understanding of the relations themselves, the understanding of the different ways in which the relationship between quantities can be expressed, and the development of problem-solving strategies.

Keywords: textual tasks, comparison tasks, comparison relations, language consistency

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