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Interactions among different types of mathematical knowledge of body volume in Mathematics textbooks²

Extended summary

The research deals with the content analysis of mathematics textbooks for the fourth grade of elementary school for measuring body volume. As the mathematics textbook is a fundamental book for learning and supporting teachers in their planning and organization of the teaching process, we consider it important for introducing and forming the concept of body volume and constructing the knowledge of students who have the characteristics of conceptual understanding and procedural flexibility when measuring body volume.

In the Theoretical framework of the research, we dealt with the concept of measuring body volume in mathematics classes, students' understanding of the concept of body volume, units of measurement - individual and composite - and the transition from composite units to multiplication. We monitored determining accurate measurements of body volume (reasoning about the procedure for measuring body volume) as a learning path, from counting individual units and continued with the repetition of individual units, the formation of composite units, the visualization of space filling, i.e. construction of a geometric body (in our case, a cuboid and a cube) based on the data on the lengths of the edges of the geometric body, and we ended

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2 The research represents the results of the modified and, in the theoretical segment, amended Master final paper *Početno razumevanje pojma zapremine učenika četvrtog razreda osnovne škole [Initial understanding of the concept of volume among the fourth-grade primary school pupils]*, defended at the Teacher Education Faculty, University of Belgrade on July 13, 2020 (mentor Olivera Đokić, PhD, associate professor).

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with developing the ability to reason about non-cubic units. Mental processes – forming and using mental models, spatial structuring, locating units in arrays and organizing space by composite units – underlie thinking about volume measurement (and area, including the previously measured length from which both area and volume are derived).

In terms of terminology, we defined two types of knowledge – conceptual and procedural – and offered exemplary mathematical tasks, and then we discussed the theoretical concepts of interactions between different types of mathematical knowledge. In our opinion, textbooks play a key role in the stated interactions, as well as in forming the concept of volume when a textbook is used for teaching and learning. For this reason, we created a list of indicators derived from a wider research of Tan Şişman (2010) on different types of knowledge in mathematics tasks related to measurement. The list of indicators helped us in the selected unit of analysis – textbook mathematics task – recording the frequency of the requirements referring to conceptual understanding and procedural knowledge in measuring body volume.

The aim of the research was to analyse two types of knowledge, conceptual and procedural, in mathematics textbooks on the topic of body volume. Based on the research on student achievement in different types of knowledge when measuring length, area, and volume (Tan Şişman, 2010), the list of indicators helped us to identify opportunities for learning in mathematics textbooks and different types of knowledge on body volume that are required in the tasks for the pupils of the fourth grade of elementary school. Encouraged by the fact that, according to the new initial instruction curricula, future fourth-graders will learn about body volume as compulsory content in mathematics classes, and by the results of the research conducted by Hurrell (Hurrell, 2021) and Hong and Runnalls (Hong, Runnalls, 2020), we formulated the following research questions:

1. Analyse the manner in which textbook topics related to body volume have been introduced and structured.
2. Determine the interaction between two types of knowledge – conceptual and procedural – in mathematics textbooks according to indicators of the specific type of knowledge.

In the paper, we applied the descriptive method and technique of content analysis of the interactions between different types of knowledge in mathematical tasks. The measuring instrument is a protocol of indicators that represents an overview of the indicators used for monitoring the presence of the elements of conceptual understanding and procedural knowledge in the textbook mathematical tasks. The research sample consisted of mathematics textbooks for the fourth grade of elementary school that were in use during our research, in the school year 2019/20, and whose publishers had approved textbooks in line with the curriculum for the third grade, assuming that they will also have textbooks for the fourth grade. The research sample included the textbooks from publishing houses: Vulkan, Klet, Freska, Kreativni centar, Novi Logos, Eduka, JP Zavod za udžbenike and BIGZ školstvo. The textbooks were analyzed as textbook sets, which, depending on the publisher, consist of only the textbook, which is also a workbook, or a textbook and a workbook, as a textbook set. Considering that the publishing houses Vulkan and Freska did not have textbooks written according to the curriculum for the fourth grade of elementary school, it was not possible to include their sets of textbooks in the

research. The publishing houses Novi Logos and Eduka, in their textbooks written according to the curriculum which was in force at the time, did not include the content of body volume, which is justified because at that time these were the so-called optional contents (it was left up to the teacher to choose whether to introduce and form the volume of the body or not), while the current curriculum classifies them as mandatory teaching contents in the fourth grade of elementary school. In the textbooks of the publishing houses Klet, Kreativni centar, JP Zavod za udžbenike and BIGZ školstvo, there were the contents on body volume, and we investigated the relationship between conceptual and procedural knowledge in them. The basic results of the research show that the indicators that are oriented towards identifying conceptual understanding showed a significantly lower frequency compared to the indicators of the procedural knowledge. We see the reason for this in the initial formation of the term body volume. Here are the main results of the research.

Structuring the textbook topics on body volume. The authors of all textbooks begin with introducing the concept of volume, and then introduce the units of volume measurement, followed by the calculation of the body volume - cuboid and cube. The publishers differ in terms of the number of the units of measurement, as well as in the structure and sequence of the textbook units. Two important elements were identified in our textbook analysis: 1) Unlike other publishing houses, Klet does not have elaborated elements for filling the space with unit cubes that lead to the idea of the units of measurement, individual and composite, nor the transition from composite units to multiplication, i.e., calculating the body volume of the cuboid and the cube, and 2) Klet and JP Zavod za udžbenike first dealt with the volume of the cube, and then of the cuboid, which can affect the mental processes of abstraction and generalisation, specialisation that the cube is the cuboid whose all sides are rectangles.

Interactions between two types of knowledge in mathematics textbooks. *Klet* – Based on the analysis of the textbook set, we found that out of the total number of the identified indicators that show two types of knowledge (86), as many as 81 (94.19%) indicate the development of procedural knowledge, while only 5 (8.81%) indicators show the development of conceptual understanding. We can conclude that this textbook set is far more oriented towards the development of procedural knowledge than conceptual understanding.

Kreativni centar – While analysing the topic of body volume in this publisher's textbook, we observed a somewhat higher percentage of conceptual understanding indicators than in all other textbooks of different publishers. However, when we analysed the entire textbook set, the percentage became slightly lower. According to our results, out of 42 identified indicators, 27 (67,29%) indicate procedural knowledge, while the remaining 15 (35,71%) indicate the development of the conceptual understanding. Our conclusion is that in this textbook set, and in terms of the body volume, attention is paid to developing both types of knowledge.

JP Zavod za udžbenike – Out of 65 identified indicators of the two types of knowledge, 55 (84,62%) indicate the development of the procedural knowledge on body volume, while the remaining 10 (15,39%) point to the development of the conceptual understanding. Our conclusion is the focus in this textbook set is on developing procedural knowledge, rather than on conceptual understanding.

BIGZ školstvo – We identified 37 indicators of the two types of knowledge, out of which 33 (89,19%) indicate the development of the procedural knowledge, and only 4 (10,81%) the development of the conceptual understanding of body volume. Our conclusion is that in this textbook set there is a far smaller focus on the development of conceptual understanding compared to procedural knowledge.

Our conclusions point to a concern regarding the structure of the textbook topics on body volume and the interaction between the two types of knowledge in the fourth-grade textbooks which are actually already out of date. We also offer questions for further research regarding the current textbooks used in Serbian schools since the school year 2020/21.

Based on the results of this research and the results of Hong and Runnalls (Hong, Runnalls, 2020) we proposed an elaborated possible model of learning related to introducing/forming the concept of volume in mathematics textbooks. We suggest that textbooks should contain the proposed learning phases, with tasks that will lead pupils to the conceptual understanding of the concept of body volume, without neglecting procedural knowledge and including procedural fluency.

With the presented model of Hong and Runnalls, we opened questions for further research on different types of knowledge when measuring body volume. The analysis of the current new curricula for the third and fourth grades and their interpretations in the form of mathematics textbooks, as well as the comparison of the results of two consecutive TIMSS research cycles (2019 - 2023) would be a further direction of research, especially if we bear in mind that a decline was identified in the last testing of the fourth grade pupils, though not statistically significant for the field of geometry and measurement (Đerić et al., 2021). Our students who were tested in the TIMSS 2019 cycle learned according to the fourth-grade mathematics curriculum in force at the time, and the textbooks that make up our research sample, while for the current TIMSS 2023 cycle they learn according to the modified current curriculum and the resulting new mathematics textbooks. Given that volume was not a compulsory part of the fourth-grade mathematics curriculum at the time our content analysis was conducted, we could say that the content structure in the textbooks is not particularly relevant either, as it is not determined by clear outcomes and instructions in the curriculum. This paper, therefore, would have a significantly greater value if the subject of the analysis were the textbooks that are currently in use. However, we consider this paper significant and justified from the aspect of methodological discussion and reasoned explanations, even though our results can already be considered out of date, as in the meantime new curricula and new textbooks with new structures of body volume topics have come into force. Continuous research and development of mathematics textbooks is a more than necessary link in the mathematics education research in Serbia (Jelić, Đokić, 2017). The very indicators that we deductively extracted from the broader research of Tan Şişman (2010) are potential opportunities for learning and teaching in the classroom, even without the use of mathematics textbooks. Therefore, we consider it important that the development of pupils' spatial competence on body volume is formed based on the results of this and similar research in mathematics education in Serbia.

Keywords: volume measurement, conceptual and procedural knowledge, mathematics textbook, curriculum, Serbia

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