

Review of the book

***DIVERSIFYING MATHEMATICS TEACHING: ADVANCED EDUCATIONAL CONTENT AND METHODS FOR PROSPECTIVE ELEMENTARY TEACHERS***

***Written by Sergei Abramovich (State University of New York, Potsdam, USA) and published by World Scientific Publishing Co. (Singapore, 2017), 268 pp. ISBN 978-981-3208-90-2***

The international academic community expects the future to be a lifetime in which significant discoveries and appropriate improvements are expected which should significantly increase the quality of life. These inventions and innovations will be realized by children who are now born. In the near future, they will have to take care of problems which we are not familiar with at the present time. Therefore, nowadays children should acquire knowledge and develop skills and abilities that would allow them to resolve many contemporary paradigms in philosophy, science, and technology. Our school system should prepare children to be able to recognize and understand current and future problems in those domains. As current knowledge is not sufficient and since the society would have to be successful in resolving situations encountered in the future, the school system should develop strategic competence and adaptive reasoning in children besides cognitive abilities and procedural skills (National Research Council, 2001). Because mathematics and grammar are the only school

subjects that strongly stimulate the development of the aforementioned proficiencies, it is quite natural to ask the question: *What needs to be done to achieve better quality mathematics education of primary school students?*

The school system that strives to prepare students to recognize, understand, and resolve the variety of situations and problems is very complex and diverse. How successful the functioning of this system will be can be influenced indirectly: by educating high-quality elementary school mathematics teachers who can deal with the diversity of the complexity of the system.

How can you diversify mathematics teaching in a teacher education course? According to Sergei Abramovich, the author of the book “Diversifying mathematics teaching: Advanced educational content and methods for prospective elementary teachers” (Singapore, World Scientific, 2017, 268 pages), the diversification of teaching stems from the diversity of teaching methods available, provided that one has a robust

conceptual knowledge of mathematics. This position is in full agreement with the modern-day perspective on the importance of the content knowledge by teachers who “must know the mathematical content they are responsible for teaching not only from a more advanced perspective but beyond the level they are assigned to teach” (Baumert et al., 2010, p. 137).

The diversity of teaching methods that the book presents is informed through continuous integration of educational psychology ideas by John Dewey (reflective inquiry as a problem-solving method), Lev Vygotsky (mediation of thinking by tools and signs), and Max Wertheimer (insight vs. Einstellung effect as a negative affordance of past experience) with ancient and classic mathematical ideas presented in an Egyptian papyrus roll (origins of the theory of congruence in geometry), writings by Archimedes (genesis of computational experiments), Blaise Pascal (birth of probabilistic reasoning), and Leonhard Euler (integer partitions as a milieu for diagrammatic representations in

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arithmetic). The role of manipulative materials and digital tools including spreadsheets, The Geometer's Sketchpad, and Wolfram Alpha is shown to be critical in this integration of powerful ideas.

Whereas Abramovich's teaching experience is framed by his interaction with prospective elementary teachers of United States and Canada, the book, in addition to a number of educational documents of the two North American countries, is informed by mathematics standards of Australia, England, Chile, Japan, Korea and Singapore. The book promotes the notion of students' questions as the main pivot of their learning of mathematics. Abramovich justifies such perspective by sharing a comment by one of his own students, an elementary school teacher candidate: "If a student asks why and a teacher cannot explain how something has come to be, the student loses all faith and interest in the subject and respect for the teacher". Towards this end, two conceptually distinct types of questions are considered: questions seeking information and questions requesting explanation. It is important for prospective teachers not only to distinguish between the two types of questions, but to learn different ways of answering them. The book emphasizes the importance of mathematical experiments, historical perspectives, the use of counterexamples, multiple ways of addressing inquiries, and the need to motivate the introduction of a new concept by demonstrating the deficiency of an existing one. In that way, the book shows not only how to teach both basic and advanced mathematical content, but it explains why the diversity of teaching methods provides an effective approach to stu-

dents' understanding of "the nature of knowledge in the discipline – where it comes from, how it changes, and how truth is established" (Ball, Thames, and Phelps, 2008, p. 393).

The book introduces what the author refers to as the W<sup>4</sup>S (we write what we see) Principle and warns about the duality of its affordances. Using grade-appropriate examples, the book demonstrates that the W<sup>4</sup>S Principle might lead learners astray in the absence of conceptual understanding. For example, only when we see that the same units are used to create the frames of reference for comparing quantities, that we can write  $3 > 2$  or  $1/3 < 1/2$ . Likewise, in the case of comparing non-unit fractions, like  $3/5$  and  $4/7$ , using area model for fractions, when dividing a rectangle (or circle, for that matter) as the unit into fifths and sevenths in only one direction (either vertically or horizontally), the outcome of comparison may be contradictory as the visual perception depends on the accuracy of drawing. This, in turn, motivates the introduction of two-dimensional area model for comparing fractions at the pre-operational level (and explains the immaturity of a pizza-like unit, often popular among elementary teacher candidates whenever fractions are concerned).

The book aims at preparing future elementary teachers to deal with mathematical situations that originate at the basic level and, through allowing students to ask questions, also evolve, as encouraged by National Mathematics Curriculum in England, towards exploring and solving problems "with increasing sophistication" (Department for Education, 2013, p. 1). Examples of that kind include discovering Pick's and Euler's

formulas by starting from constructing different shapes on a geoboard, adding consecutive natural numbers as a premise of finding the number of trapezoidal representations for a given integer, decomposing an integer into a sum of two integers as a motivation for understanding Ferrers-Young diagrams – advanced versions of "tape diagrams" (Common Core State Standards, 2010), recording a sample space when tossing two coins as the preparation for experimentally calculating probability of irreducibility of a fraction with randomly selected numerator and denominator. Another conceptually powerful hands-on exploration presented in the book deals with the context of the classic triangle inequality – the sum of any two sides of a triangle is greater than the third side. It is shown how the exploration, stemming from an activity with straws recommended for second graders in the state of New York, can be gradually extended in the classroom of interested teacher candidates to develop, in particular, a sequence representing the number of ways to construct an integer sided triangle with a given longest side using unit-length straws of three colors. This sequence – 10, 300, 9405, 271701, 7586055, ... – has been included in the Online Encyclopedia of Integer Sequences (<http://oeis.org/A278037>), a rich source of mathematical knowledge, referencing the paper (Abramovich, 2016) where such an extension was originally suggested.

The book can be a very helpful source of information supplementing a basic textbook for a methods course for prospective elementary school teachers. As written in the preface, "the book includes material that the author would have shared

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with teacher candidates had there have been more instructional time than a 3-credit hour master's level course "Elementary Mathematics: Content and Methods" provides". The book can also be used with secondary school mathematics teacher candidates demonstrating the rich interplay of diverse ideas within the entire pre-college mathematics curricula. Finally, the book can be recommend-

ed for a doctoral level mathematics education programs as its content may prompt many ideas to be further explored under the disciplined inquiry umbrella.

To conclude this review, we express our conviction that the book of this productive author will significantly contribute to a better mathematical education of the primary school teachers.

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