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The Equals Sign – the Problem of Early Algebra Learning and How to Solve It

Extended summary

One of the most important concepts and symbols in mathematics is the *equals sign*. The correct understanding of this concept is crucial for learning and understanding other mathematical content, especially algebra. The paper specifically looks at the difficulties related to the misunderstanding of the equals sign which stem from the precarious foundations on which the said concept is formed in arithmetic learning where students understand this sign as a sign that signifies the command “calculate” or “ascertain”. To understand the equals sign it is vitally important to understand it not only operationally, but also to develop the relational meaning of this symbol. Understanding the sign “=” only operationally results in students’ difficulties in understanding its relational meaning. Bearing this in mind, the authors of the paper put forward and elaborate on the idea that learning and formation of the equals sign at early school age should be based on learning situations in which pupils can model real situation and solve tasks in realistic contexts. In this manner, the problems in the realistic context reflect the real life situations indicating the essence of the equals sign concept – as a sign expressing equivalence. The paper presents a methodological approach to learning based on modelling the real-life context situations. This approach should solve the problem of misunderstanding the concept of the equals sign. In modelling the real-life context situations we emphasize length and balance modelling because these models enable the easiest understanding and identification of equivalency as a basis for forming the concept of the equals sign in mathematics. The presented methodological approach was experimentally tested in the research the goal of which was to

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determine whether this methodological approach contributes to a better understanding of the equals sign. The research was conducted by means of an experiment with parallel groups, on a sample of the fourth-grade pupils (N=257). The effects of the methodological approach were tested in this experiment. The program used with the experimental group consisted of the specially prepared lessons (25) in which the real-life context modelling was applied, whereas the pupils in the control group were taught in the traditional way by using the approved mathematics textbook for the fourth grade of primary school. The obtained results showed that the relational understanding of the equals sign can be developed by using problem tasks expressing the real-life context situations. The results also indicate that pupils' ability to understand properly the equals sign can be enhanced by using carefully designed mathematics content based on realistic situations close to pupils' knowledge and rooted in their own experience. The research showed that the implementation of realistic contexts can improve the process of learning the content of arithmetics and algebra, which in turn will create a sound foundation for a proper formation of not only the equals sign, but also other mathematical concepts. Moreover, the modelling of realistic situations enables pupils to develop their knowledge of mathematics gradually, move, step by step, into the area of formal mathematical symbolism, and avoid potential problems or misconceptions regarding the content of algebra.

Keywords: contextual approach, equals sign, early algebra, Realistic Mathematics Education, student achievement

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