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Mental Arithmetic – Subtraction Strategies

Extended summary

Mental strategies can be defined as "clever" calculation methods based on understanding the fundamental characteristics of the number systems and arithmetic operations, as well as on the well-developed sense for numbers (Verschaffel et al., 2007). These strategies differ from the algorithm of numeric calculations in the following manner (Linsen, et al., 2015): 1) mental calculation involves numbers, not digits; 2) there is no single correct way of doing calculations; (3) numbers usually appear in a horizontal line, and (4) mental calculations have fewer written notations, or have none at all. As many studies indicate, mental calculation involves a wide spectrum of strategies (Peltenburg et al., 2012; Selter et al., 2012; Torbeyns et al., 2009; Verschaffel et al., 2007). A dominant characteristic of mental arithmetic is its *strategic flexibility in terms of selecting a strategy relative to the characteristics of numbers in a mathematical task* (Verschaffel et al., 2010; Blöte et al., 2001. and others).

The goal of the research was to examine pupils' skills for doing mental subtraction, and determine whether they possessed a strategic flexibility during the process. Mental subtraction involves calculation procedures in which the pupils use numbers (not digits), without writing down partial results (Linsen, et al., 2015). Strategic flexibility implies selecting specific strategies depending on the characteristics of the numbers in a mathematical task (Blöte et al., 2001). A descriptive method and a standardised interview with open-ended questions were used in the research. The research was conducted on a sample of 66 third-graders from two primary schools in Belgrade. The respondents were interviewed individually and an audio recording of every interview was made as well. The pupils had to calculate the value of five expressions (605 – 98, 107 – 95, 178 – 96, 95 – 88, 87 – 49). The content of the interviews was analysed by using a deductive method. The following categories were used in the analysis of the obtained data:

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1) a standard procedure of written (in digits) calculation; 2) decomposition strategies (Selter et al., 2012; Verschaffel et al. 2007); 3) sequencing strategies (Selter et al., 2012; Verschaffel et al., 2007; Torbeyns et el., 2009b); 4) a combined method (Fuson et al., 1997) 5) compensation strategies (Selter et al., 2012; Peltenburg et al.; 2012; Verschaffel et al. 2007): a) subtracted "round" number, compensation b) balancing; 6) indirect adding (Torbeyns et al., 2009b); 7) indirect subtraction (Torbeyns et al., 2009b). Though calculating by using digits was mentioned as the first category for collecting the data, it should be noted that the standard procedure of written calculation (using digits) is not a mental arithmetic strategy because it involves digits, not decimal sums.

The obtained results indicate that pupils predominantly (compared to others) use the strategy of digital calculation, and more rarely (compared to others) the sequencing strategy ($\chi 2$ (5. 326)=210.761. p=0.000). However, if all mental arithmetic strategies are viewed as a single category, statistically the pupils were more inclined to choose the mental arithmetic strategies than the algorithm of digital calculation ($\chi 2$ (1. 326)= 6.491. p = .011). It remains unclear whether this is an indicator of the lack of conceptual understanding of the numerical system and its rules (Blöte et al., 2000) or the pupils simply choose the strategy they are familiar with (Baranes et al., 1989) and think that they are expected to do so (Torbeyns et al., 2008; Imbo & Vandierendonck, 2006). The analysis of the number of strategies used by individual pupils in different examples also shows that pupils lack flexibility in doing calculations. More than a half of pupils, exactly 36, (54.55%), use only one strategy in all examples. Two strategies were used by 19 pupils (28.79%), three strategies by 9 pupils (13. 64%), one pupil (1.52%) used four strategies and one pupil (1.52%) used five subtraction strategies. One third of pupils (22 pupils, 33.3%) used only the digital strategy (written calculation) in all examples. The comparison of the pupils' success in using the algorithm of digital calculation with their success in using mental arithmetic shows that the pupils are statistically much more successful when they use mental arithmetic strategies ($\chi 2$ (1. 326)= 12.54,1 p=.000). In terms of statistics, the number of pupils' errors was much higher when they were using the algorithm of digital calculation.

Given that the pupils were mostly using the algorithm of digital calculation in trying to do a calculation without a paper and pencil, we can conclude that they are not capable of doing mental calculations and they lack a strategic flexibility in the process of doing calculations. We believe that the implementation of the mental arithmetic strategies is an indicator of the pupils' conceptual understanding of the decimal system and mathematical operations. Therefore, the students lacking this understanding did not choose the mental arithmetic strategies because they were not able to finish the whole procedure (Blöte et al., 2000). In our opinion, and without questioning the importance of learning and understanding standard algorithms, mental arithmetic strategies should be included in the early primary curriculum.

Keywords: mental arithmetic, subtraction strategies, strategic flexibility.

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