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**Original
research paper**

Paper received: Jun 17 2025
Paper accepted: Nov 27 2025
Article Published: Jan 20 2026


Developing early algebraic thinking through visually presented growing patterns in lower primary school students

Extended summary

Developing early algebraic thinking is one of the primary tasks of teaching mathematics in the lower grades of primary school and a key aspect of algebraic education, but also a major challenge for all practitioners in education and teaching. The aim of the paper is to investigate whether, with the help of visually presented growing patterns, an abstract level of thinking can be reached, namely, whether students can determine the elements of the pattern at distant positions and whether and how many of them can independently reach the level of generalization. The increasing sequence (pattern) has characteristics that make it unique and ideal for supporting the development of students' functional thinking. Such a sequence involves two variables, the dependent variable being some measurable aspect of the pictorial pattern of the object that coordinates with an indexing or counting system that is the independent variable and defines the position of the figure in the sequence. This allows geometric patterns in the form of increasing sequences to provide an opportunity for the conceptual understanding of the variables. Students can describe both verbally and numerically how the sequence (pattern) increases and how it translates into the symbolic language of algebra.

The research, based on the testing technique, was conducted on the population of the third- and fourth-grade primary school students. The sample was selected from primary schools in Zlatibor District. The sample consisted of two subsamples, third-grade students (106 students) and fourth-grade primary school students (146 students). The research instrument

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was a three-item test, the sequence models in the tasks were taken from Markworth (Markworth, 2010), with minor corrections and adjustments. The questions in the tasks ranged from figurative to numerical reasoning, and the final question in each of the three tasks referred to a symbolically expressed generalization.

Based on the results obtained in the research, we can conclude that the students in this age group, although they have the ability to use the visualization of a sequence, identify the rule and the distant members, still fail to identify the general member of the sequence. This indicates that students have not mastered generalization in algebra nor do they fully understand the concept of a variable, and that figurative reasoning, or the visual nature of the pattern itself, leads students to notice the rules and understand the functional dependence of member in the sequence on the ordinal number of its position. The research shows that the way of representing functional dependence visually, with geometric ascending patterns, has great potential to make students interested in identifying a symbolic or verbal rule, as well as to provide appropriate support and challenge, which in turn leads to the success of all students. The research results confirm that the visual representation of quantitative relationships is important for building the meaning of algebraic symbols and procedures. Consequently, geometric patterns, rather than numerical ones, should be used in the teaching and learning of mathematics content, given that tasks with geometrically given patterns enable students, with the help of visualization, to be familiarized with the concept of functional dependence and the ways of expressing it.

Keywords: growing patterns, visualization, functional dependence, generalization

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